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1	The Firefighter Algorithm for Optimization Problems
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8 Abstract

9 This paper presents the Firefighter Optimization (FFO) algorithm as a new metaheuristic for optimization problems that stems inspiration from the collaborative strategies often deployed by 10 firefighters in firefighting activities. Such strategies include adaptive response to changing 11 conditions, coordination among multiple firefighters (i.e., agents) to converge on a common goal, 12 balancing exploration and exploitation by maintaining diversity within the search space and 13 adapting its parameters to navigate complex landscapes. To evaluate the performance of FFO, 14 extensive experiments were conducted, wherein the FFO was examined against 13 commonly used 15 optimization algorithms, namely, the Ant Colony Optimization (ACO), Bat Algorithm (BA), 16 Biogeography-Based Optimization (BBO), Flower Pollination Algorithm (FPA), Genetic 17 Algorithm (GA), Grey Wolf Optimizer (GWO), Harmony Search (HS), Particle Swarm 18 Optimization (PSO), Simulated Annealing (SA), Tabu Search (TS), and Whale Optimization 19 Algorithm (WOA), and across 24 benchmark functions, as well as 10 standard functions and 4 real 20 engineering problems from the CEC 2020 suite. The results demonstrate that FFO achieves 21 comparative performance and, in some scenarios, outperforms commonly adopted optimization 22 algorithms in terms of the obtained fitness, time taken for exaction, and research space covered 23 per unit of time. More specifically, FFO ranked first in the Distance per Unit Time metric and 24 25 maintained a top 5 performance in higher dimensions (i.e., 20D and 50D).

26 <u>*Keywords*</u>: Optimization; Benchmarking; Metaheuristics.

27 **1.0 Introduction**

Metaheuristics play a large role in the domain of optimization. These algorithms have been 28 renowned for their efficacy in tackling complex and multidimensional problems that can typically 29 be beyond the reach of traditional methods [1]. Metaheuristics are distinguished by their flexibility 30 and robustness, making them particularly suitable for problems where the solution landscape is 31 rugged or poorly defined, such as those expected across diverse disciplines, ranging from 32 engineering and logistics to economics and data science [2]. For example, metaheuristics' 33 versatility enables their engineering application to optimize design parameters for complex 34 systems. In logistics, metaheuristics can help solve scheduling and routing problems. Similarly, 35 metaheuristics can prove beneficial in finance systems to optimize investment portfolios, to name 36 a few. 37

Metaheuristics can be defined as high-level strategies that coordinate simpler investigative methodologies to explore and exploit the search space efficiently [3]. These strategies are characterized by their reliance on processes that promote some form of balance between

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- 41 exploration of the search space to avoid entrapment in local optima and exploitation mechanisms
- that refine promising areas to converge toward global optima [4]. Thus, metaheuristics can be
- thought of as generic and adaptable to a broad spectrum of problems with minimal modification.
- 44 This can be advantageous to problem-specific algorithms.

Metaheuristics are broadly classified into two categories based on their approach: single-solution 45 based and population-based. Single-solution metaheuristics iteratively improve a single candidate 46 solution. These algorithms capitalize on the collective intelligence of the population to explore and 47 exploit the search space more broadly. Some such metaheuristics include Simulated Annealing 48 (SA) and Tabu Search (TS) [5]. This group of algorithms employs mechanisms to escape local 49 optima, like probabilistic acceptance of worse solutions in SA or using memory structures in TS 50 to avoid revisiting previously explored areas of the search space. The efficiency of single-solution 51 metaheuristics is tied to their ability to fine-tune a solution through mechanisms like adaptive 52 neighborhood searches and intensification strategies that home in on promising regions of the 53

54 search space [5].

⁵⁵ On the other hand, population-based metaheuristics evolve a group (i.e., population) of solutions

- to leverage interactions within this group to explore and exploit the search space collectively [6].
- 57 Some examples under this group include Genetic Algorithms (GA) and Particle Swarm 58 Optimization (PSO). Such algorithms can maintain diversity within the population and avoid
- premature convergence to suboptimal solutions. For instance, GA employs operators such as 59 crossover and mutation to introduce variability and ensure robust exploration, while PSO mimics 60 social behaviors of swarms [7]. These two examples can balance exploration and exploitation 61 through the dynamic adjustment of particle velocities based on individual and collective 62 experiences. This population-based approach enables these algorithms to effectively navigate 63 complex, multimodal optimization landscapes by simultaneously exploring multiple regions of the 64 search space, increasing the likelihood of finding a global optimum. It goes without saying that 65 hybrid approaches that combine elements from both single-solution and population-based 66 metaheuristics have emerged as a means to leverage the strengths of both strategies [8]. These 67
- ⁶⁸ hybrid methods often incorporate mechanisms like adaptive parameter control, multi-stage search
- ⁶⁹ processes, and cooperative co-evolution to enhance performance on a wide range of optimization
- 70 problems [9].

Similarly, metaheuristics can also be classified based on their source of inspiration. Some metaheuristics are nature-inspired algorithms, wherein they are inspired by natural phenomena, biological processes, or behaviors observed in animals/plants. These nature-inspired algorithms often mimic survival mechanisms, evolutionary processes, or social behaviors. For example, GA, PSO, and Ant Colony Optimization (ACO) are representative of such classification. Then, some

- ⁷⁶ metaheuristics draw inspiration from human behaviors, the physical world, and societal structures.
- For instance, both TS and SA incorporate actions and behaviors seen in humans (i.e., memory)
- and physical processes (i.e., annealing in metallurgy) [10].

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Despite their versatility and ease of use, metaheuristics can suffer from challenges [11]. One such 79 challenge is drawing a balance between exploration and exploitation capabilities. These 80 capabilities can be crucial for avoiding premature convergence and ensuring the global optimum 81 is reached. Moreover, the stochastic nature of these algorithms often requires multiple runs to 82 achieve consistent results, which can be computationally expensive [12]. Fortunately, the field of 83 metaheuristics continues to grow in response to addressing increasingly complex problems 84 [13,14]. One notable trend is the hybridization of metaheuristic algorithms, where two or more 85 distinct strategies are combined to exploit their complementary strengths. For instance, hybrid 86 algorithms might combine one algorithm's explorative power with another's intensive exploitation 87 capabilities. Such hybridization can potentially yield solutions that are both diverse and precise. 88

Further, recent studies have proposed novel metaheuristic algorithms such as the Improved 89 Crowding Particle Algorithm (I-CPA) and the Tree Seed Algorithm (TSA) [15]. For instance, I-90 CPA has been applied to optimize engineering design problems with multi-objectives and complex 91 constraints, improving the balance between exploration and exploitation [16]. Similarly, TSA, 92 inspired by tree seed dispersal mechanisms, has been successfully used in solving scheduling and 93 resource allocation problems [17]. These advancements highlight the ongoing evolution and 94 diversification of metaheuristic algorithms and showcase their adaptability to various domains. In 95 addition to these recent advancements, other metaheuristics such as the Harris Hawks Optimization 96 (HHO) and the Arithmetic Optimization Algorithm (AOA) have gained attention due to their 97 robust performance across a variety of domains [18,19]. HHO, inspired by the cooperative hunting 98 strategy of Harris hawks, has been applied to challenging problems, including engineering, 99 medical, data mining and clustering [20]. AOA, based on arithmetic operations, has also shown 100 101 success in solving complex mathematical and engineering optimization problems. These emerging algorithms further underline the potential of metaheuristics in addressing diverse optimization 102 challenges, contributing to a growing repository of tools designed to tackle specific industrial and 103 research needs [19]. 104

Still, as computational challenges grow and the need for efficient optimization strategies 105 intensifies, the role of metaheuristics becomes increasingly important and warranted. The ability 106 of metaheuristics to adapt and provide feasible and efficient solutions. The above motivates this 107 work wherein we propose a novel algorithm, Firefighter Optimization (FFO), for optimization 108 problems that could be applied in various areas (e.g., a general purpose algorithm). FFO is 109 motivated by the strategies and tactics employed by firefighters, such as the dynamic distribution 110 of resources, adaptive responses to evolving conditions, and coordinated efforts among multiple 111 firefighters (agents) to achieve a unified objective. FFO also balances exploration and exploitation 112 by maintaining diversity within the search space and adjusting its parameters to navigate complex 113 optimization challenges. A number of comparative experiments, supplemented with various 114 metrics, were carried out to validate the effectiveness and competitiveness of FFO. More 115 specifically, the performance of FFO and the other selected algorithms was examined across over 116

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- 600 tests on a wide range of benchmark and test functions. Our experimental results demonstrate
 the effectiveness and competitiveness of FFO compared to state-of-the-art algorithms.
- 119 The rest of the paper is organized as follows: Section 2 presents a description of FFO and explains
- each component in detail. Sections 3 and 4 describe the aforementioned algorithms and 24
- commonly used benchmarking test functions for various complexity levels. Finally, Sections 4 and
- 122 5 conclude the paper with a presentation of our comparative results and conclusions/findings
- 123 learned from our analysis.

124 **2.0 Description of the Firefighter Optimization (FFO) algorithm**

125 This section describes the FFO in more detail (see flowchart in Fig. 1). We start with a general 126 description and then dive into a more detailed analysis of FFO's functions.

127 2.1 General description

- The Firefighter optimization (FFO) algorithm is inspired by the strategies and tactics used by 128 firefighters to combat fires in real-world scenarios. In the face of a fire, firefighters must 129 strategically allocate resources, decide when to focus on extinguishing the flames, and when to 130 protect specific areas or perform rescue operations. More specifically, this optimization technique 131 draws from various aspects of firefighting, including the dynamic allocation of resources, adaptive 132 response to changing conditions, and coordination among multiple firefighters (i.e., agents) to 133 achieve a common goal. Further, the FFO algorithm is designed to balance exploration and 134 exploitation by maintaining diversity within the search space and adapting its parameters to 135 navigate complex optimization landscapes. See Table 1 for a comparison of this algorithm against 136
- 137 other commonly used in optimization.
- Initialization: The algorithm starts by randomly initializing a population of agents within the
 specified bounds of the search space. Each agent represents a potential solution to the optimization
 problem.
- Evaluation: Each agent's position is evaluated using the objective function, which measures the fitness or quality of the solution. The best agent, i.e., the one with the lowest fitness value for minimization problems, is identified as the global best solution.
- Adaptive Local Search: The FFO algorithm employs an adaptive local search mechanism to refine the positions of the agents. This process involves generating perturbations around the current position of an agent and evaluating the new positions. The perturbations are adjusted adaptively based on the agent's performance and the iteration count. This local search helps agents escape local optima and explore the solution space more effectively.
- 149 Crossover and Mutation: To enhance diversity and promote exploration, the FFO algorithm 150 incorporates crossover and mutation operators. The crossover operator allows agents to exchange
- information, creating new solutions by combining parts of two parent agents. The mutation

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- operator introduces random changes to an agent's position, providing an additional mechanism to
 explore new areas of the search space.
- Perturbation Mechanism: When agents fail to improve over a certain number of iterations, a perturbation mechanism is triggered. This mechanism mimics the adaptive response of firefighters to changing conditions. Agents undergo a larger perturbation, guided by the global best agent, to move towards potentially better regions of the search space. The intensity of the perturbation increases with the number of unsuccessful iterations, allowing the algorithm to escape stagnation end continue searching for antimal colutions.
- and continue searching for optimal solutions.
- Adaptive Step Size: The step size used in the local search and perturbation mechanisms is adaptively adjusted based on the algorithm's progress. If the algorithm detects stagnation, the step size is increased to encourage exploration. Conversely, if the algorithm is converging towards a solution, the step size is reduced to fine-tune the search and improve solution accuracy.
- Cooling Schedule: The FFO algorithm incorporates a cooling schedule inspired by simulated annealing. The temperature parameter, which controls the acceptance probability of worse solutions during the local search, is gradually reduced over iterations. This allows the algorithm to initially explore more freely and then gradually focus on exploitation as it approaches convergence.
- 169 Termination Criteria: The algorithm runs until one or more termination criteria are met. These
- 170 criteria can include reaching a maximum number of iterations, exceeding a predefined number of
- iterations without improvement, or achieving a target fitness value. The termination criteria ensure
- that the algorithm does not run indefinitely and provides a solution within a reasonable time frame.

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Fig. 1 Flowchart of FFO

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- 175 *2.2 Detailed description*
- 176 A more detailed description of FFO's functions is provided herein.
- 177 Initialization (__init__)
- The initialization function (__init__) of the FirefighterOptimization class sets up the algorithm's parameters, agents, and initial conditions for optimization. These parameters include the objective
- 180 function, dimensions of the problem, number of agents, maximum iterations, no-improvement
- 181 limit, and bounds for the search space. Additional parameters stemming from existing algorithms
- 182 for crossover, mutation, simulated annealing, and perturbation control are also set up. The agents
- 183 (i.e., solutions) are initialized randomly within the specified bounds, and the best global agent and
- 184 fitness are identified at the start.

185 Initialization Process:

186 The process can be broken down into the following steps:

187	Parameter Setup
188	Agents Initialization: Agents are randomly distributed within the bounds:
189	Best Agent Identification: The initial best global agent and its fitness are determined:
190	Other Parameters: Additional parameters like step size, mutation rates, and counters are initialized:

191 Agent Evaluation (evaluate_agents)

192 The evaluate_agents function assesses the fitness of each agent within the initiated population.

- 193 This function updates the best global fitness and agent if/when a better solution is found. This
- 194 function calculates the fitness of each agent based on the objective function and updates the global
- best agent if an improved solution is identified. This evaluation process also guides the algorithm's
- search process toward better solutions. Mathematically, the evaluation involves computing the
- ¹⁹⁷ objective function for each agent and identifying the agent with the minimum fitness value.
- Objective Function, f(x): The function to be minimized and is evaluated for each agent x_i . 198 <u>Fitness Calculation</u>: For each agent x_i , the fitness is $f(x_i)$. 199 Best Fitness Update: The global best fitness and agent are updated if a new minimum is found. 200 **Evaluation Process** 201 The evaluation process can be broken down into the following steps: 202 203 **Fitness Calculation Best Agent Identification** 204 Return Fitness Values 205

206 Agent Update (update_agents)

The update_agents function is responsible for evolving the population of firefighters (i.e., agents) to maintain diversity through crossover, mutation, and perturbation operations. This function involves modifying agent positions in the solution space to explore new regions. As inspired by genetic algorithms, this function applies crossover to combine traits from different agents,

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- ²¹¹ mutation¹ to introduce random changes, and perturbations to escape local optima. Mathematically,
- the update process includes the following key components:

213	Crossover Operation
214	Mutation Operation
215	Perturbation Operation
216	<u>Update Process</u> : The update process involves the following steps:
217	Evaluate Agents: The fitness of agents is then evaluated to update the global best fitness achieved:
218	Crossover and Mutation: Each agent undergoes crossover and mutation based on set probabilities:
219	Perturbation: If no improvement is observed for an extended period, this function applies
220	perturbations:
221	Boundary Check: Agents are clipped within the bounds:
222	Trajectory and Perturbation History: Updates to agents are recorded for trajectory analysis
223	

224 Local Search (local_search)

The local search function refines an agent's position by exploring its neighborhood. This function 225 helps agents escape local optima and find better solutions. This function involves making small 226 adjustments to an agent's position to find a better solution in its vicinity. The process is guided by 227 a temperature parameter, allowing the acceptance of worse solutions early on to escape local 228 optima. As the temperature decreases, the search becomes more focused on local refinement – in 229 a similar process to controlling fires. Mathematically, local search applies perturbations to an 230 agent's position and evaluates the new positions. The acceptance of new positions is probabilistic, 231 influenced by a temperature parameter. 232

233	Perturbation
234	Temperature
235	Acceptance Probability
236	Local Search Process: The process involves the following steps:
237	Temperature Calculation: The temperature is calculated based on the iteration:
238	Local Best Initialization: The current agent is considered the local best:
239	Perturbation and Evaluation: Small adjustments are made to the agent's position, and new positions
240	are evaluated:
241	Acceptance Check: New positions are accepted based on fitness improvement or probabilistically:
242	<i>Return Local Best:</i> The refined local best position is then returned:

243 **Perturbation Application** (apply_perturbation)

Perturbation application is a strategy to escape local optima by making larger adjustments to agents' positions, which can be particularly useful when the algorithm stagnates. The apply_perturbation function introduces significant changes to agents' positions based on the global best agent when no improvement is observed. In mathematical notation, perturbation involves adjusting an agent's position towards the global best agent, scaled by an intensity factor, such that:

249 Direction Vector:

250 <u>Perturbation:</u>

¹ Further information on these operations will be provided in a subsequent section.

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251	Perturbation Process
252	The process involves the following steps:
253	Direction Calculation: The direction vector from the agent to the global best is calculated:
254	Perturbation Calculation: A perturbation is applied based on the direction vector and intensity:
255	New Position Calculation: The agent's new position is calculated:

256 **Crossover** (crossover)

The crossover function combines parts of two agents to create new agents as a means of introducing diversity into the initialized population. This genetic algorithm-inspired method helps explore new solutions by recombining existing ones. Key components in this function include:

260	Crossover Point
261	New Agents
262	Crossover Process: The process involves the following steps:
263	Crossover Point Selection: A random crossover point is selected:
264	Agent Combination: New agents are created by combining segments of the parent agents:
265	Return New Agents: The new agents are returned:

266 **Cooling Schedule** (cooling_schedule)

The cooling_schedule function adjusts the step size based on the algorithm's progress, similar to the *cooling* option in simulated annealing. The cooling schedule involves gradually reducing the step size as the algorithm progresses (in a similar manner to controlling the fire toward the later stages of firefighting). This process allows for a finer search of the solution space over time, balancing exploration and exploitation. Mathematically, the cooling schedule involves updating the step size based on the number of iterations and the no-improvement counter such that:

- 273 <u>Step Size Update:</u>
 274 The step size is reduced based on a cooling factor.
 275 <u>Cooling Process:</u>
- 276The process involves the following steps:277Step Size Adjustment: The st
 - Step Size Adjustment: The step size is adjusted based on the no-improvement counter:

279 **Execution Loop (**run**)**

278

The run function controls the main execution loop of the algorithm, where agents are updated, 280 evaluated, and the cooling schedule is applied until a termination condition is met. It also tracks 281 the best solution and its fitness across iterations. The execution loop is the core of the optimization 282 process. It iteratively updates agents, evaluates their fitness, applies the cooling schedule, and 283 checks termination conditions. This loop continues until the optimization criteria are met, ensuring 284 that the algorithm converges to an optimal solution. Mathematically, the execution loop involves 285 iterating over the update and evaluation processes while tracking the best solution. Key 286 components include: 287

288	Execution Process
289	The process involves the following steps:
290	Initialization: The fitness history and trajectory are initialized
291	Main Loop: The main loop iterates until the termination condition is met

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292 *Return Best Solution*: The best global agent and fitness are returned

293 **Termination Check** (should_terminate)

- 294 The should_terminate function determines whether the algorithm should stop based on several
- 295 conditions: maximum iterations reached, no improvement for a set number of iterations, or
- achieving a target fitness level. Thus, this function ensures that the algorithm terminates when
- ²⁹⁷ further exploration is unlikely to yield better results.
- 298 <u>Iteration Check:</u> The algorithm stops if the maximum number of iterations is reached (i.e., $k \ge \max_iter$).
- 299 <u>No Improvement Check:</u> The algorithm stops if no improvement is observed for a set number of iterations.
- 300 <u>Target Fitness Check:</u> The algorithm stops if the target fitness level is achieved.
- 301 <u>Termination Process</u>
- 302 The process involves the following steps:
- 303Condition Check: The termination condition is evaluated based on the iteration, no-improvement304counter, and best global fitness
- 305 *Return Condition*: The termination condition is returned

306 **Execution Time (get_execution_time)**

307 The get_execution_time function calculates the total runtime of the algorithm as a means to

- 308 present a measure for evaluating the time efficiency of the algorithm. This execution time is
- 309 calculated as the difference between the end time and the start time.
- 310 **Trajectory Tracking (get_trajectory)**
- 311 The get_trajectory function records the sequence of solutions explored by the algorithm, which
- 312 can be further analyzed to examine the search behavior and pathway through the solution space.
- 313 The trajectory involves recording the positions of agents over time.

314 **Total Distance Traveled (get_total_distance)**

- 315 The get_total_distance function computes the cumulative distance traveled by the algorithm in the
- 316 solution space. Such a distance can be an indicator of the algorithm's exploratory behavior and
- 317 efficiency. The total distance traveled involves summing the Euclidean distances between
- 318 consecutive positions of agents.

319 class FirefighterOptimization:

- 320 def __init__(self, objective_func, dimension, num_agents=100, max_iter=500, no_improve_limit=30, bounds=(-
- 5.12, 5.12), step_size=1.0, crossover_probability=0.5, mutation_probability=0.1, initial_temp=100.0,
- 322 cooling_rate=0.95, verbose=False, use_additional_conditions=False, target_fitness=1e-5):
- 323 self.objective_func = objective_func
- 324 self.dimension = dimension
- 325 self.num_agents = num_agents
- 326 self.max_iter = max_iter
- 327 self.no_improve_limit = no_improve_limit
- 328 self.bounds = bounds
- 329 self.agents = np.random.uniform(bounds[0], bounds[1], (num_agents, dimension))
- 330 self.best_global_agent = np.copy(self.agents[np.argmin([self.objective_func(agent) for agent in self.agents])])

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331	self.best_global_fitness = self.objective_func(self.best_global_agent)
332	self.step_size = step_size
333	self.crossover_probability = crossover_probability
334	self.mutation_probability = mutation_probability
335	self.initial_temp = initial_temp
336	self.cooling_rate = cooling_rate
337	self.mutation_rates = np.full(self.num_agents, 0.1)
338	self.no_improve_counter = 0
339	self.iteration = 1
340	self.fitness_history = []
341	self.perturbation_history = []
342	self.verbose = verbose
343	self.use_additional_conditions = use_additional_conditions
344	self.target_fitness = target_fitness
345	self.trajectory = []
346	self.start_time = None
347	self.end_time = None
348	
349	def evaluate_agents(self):
350	fitness = np.array([self.objective_func(agent) for agent in self.agents])
351	best_index = np.argmin(fitness)
352	if fitness[best_index] < self.best_global_fitness:
353	self.best_global_fitness = fitness[best_index]
354	self.best_global_agent = np.copy(self.agents[best_index])
355	self.no_improve_counter = 0
356	else:
357	self.no_improve_counter += 1
358	return fitness
359	
360	def update_agents(self):
361	self.evaluate_agents()
362	for i in range(self.num_agents):
363	if np.random.rand() < self.crossover_probability:
364	partner_index = np.random.randint(self.num_agents)
365	self.agents[i], self.agents[partner_index] = self.crossover(self.agents[i], self.agents[partner_index])
366	if np.random.rand() < self.mutation_probability:
367	self.agents[i] = self.local_search(self.agents[i], i)
368	if self.no_improve_counter > 50:
369	self.agents[i] = self.apply_perturbation(self.agents[i], 0.1 + 0.02 * (self.no_improve_counter - 50))
370	<pre>self.agents[i] = np.clip(self.agents[i], self.bounds[0], self.bounds[1])</pre>
371	self.trajectory.append(np.copy(self.agents[i]))
372	self.perturbation_history.append(self.agents[i])

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373	
374	def local_search(self, agent, index):
375	temp = self.initial_temp * (self.cooling_rate ** self.iteration)
376	best_local = agent
377	best_local_fitness = self.objective_func(agent)
378	for _ in range(10 + 5 * (self.no_improve_counter // 100)):
379	perturbation = np.random.normal(0, self.step_size * self.mutation_rates[index], self.dimension)
380	candidate = best_local + perturbation
381	candidate_fitness = self.objective_func(candidate)
382	if candidate_fitness < best_local_fitness or np.random.rand() < np.exp((best_local_fitness -
383	candidate_fitness) / temp):
384	best_local = candidate
385	best_local_fitness = candidate_fitness
386	return best_local
387	
388	def apply_perturbation(self, agent, intensity):
389	direction = self.best_global_agent - agent
390	perturbation = np.random.normal(0, intensity, self.dimension) * direction
391	return agent + perturbation
392	
393	def crossover(self, agent1, agent2):
394	crossover_point = np.random.randint(1, self.dimension)
395	<pre>new_agent1 = np.concatenate((agent1[:crossover_point], agent2[crossover_point:]))</pre>
396	new_agent2 = np.concatenate((agent2[:crossover_point], agent1[crossover_point:]))
397	return new_agent1, new_agent2
398	
399	def cooling_schedule(self):
400	if self.no_improve_counter > 50:
401	self.step_size *= 0.98
402	else:
403	self.step_size *= 0.99
404	
405	def run(self):
406	self.fitness_history = []
407	self.trajectory = []
408	self.iteration = 1
409	self.start_time = time.time()
410	while not self.should_terminate():
411	self.update_agents()
412	self.cooling_schedule()
413	self.fitness_history.append(self.best_global_fitness)
414	if self.verbose:

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415	print(f"Iteration {self.iteration}, Best Fitness {self.best_global_fitness}, No Improve Counter							
416	{self.no_improve_counter}")							
417	self.iteration += 1							
418	self.end_time = time.time()							
419	return self.best_global_agent, self.best_global_fitness, self.fitness_history							
420								
421	def should_terminate(self):							
422	if self.use_additional_conditions:							
423	termination_condition = (
424	self.iteration >= self.max_iter or							
425	self.no_improve_counter > self.no_improve_limit or							
426	self.best_global_fitness < self.target_fitness							
427)							
428	else:							
429	termination_condition = self.iteration >= self.max_iter							
430	if termination_condition and self.verbose:							
431	print(f"Terminating: Iteration={self.iteration}, No Improve Counter={self.no_improve_counter}, Best							
432	Fitness={self.best_global_fitness}")							
433	return termination_condition							
434								
435	def get_execution_time(self):							
436	return self.end_time - self.start_time							
437								
438	def get_trajectory(self):							
439	return self.trajectory							
440								
441	def get_total_distance(self):							
442	distance = 0							
443	for i in range(1, len(self.trajectory)):							
444	distance += np.linalg.norm(self.trajectory[i] - self.trajectory[i - 1])							
445	return distance							

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Table 1 Qualitative comparison between FFO and commonly used optimization algorithms

Feature/Aspect	FFO	ACO	BA	BBO	FPA	GA	GWO	HS	PSO	SA	TS	WOA
Inspiration	Firefighting	Ant foraging	Echolocation of	Biogeography	Flower	Natural	Grey wolves'	Musical harmony	Swarming	Annealing	Memory-based	Whale bubble-
	strategies	behavior	bats	concepts	pollination	evolution	hunting		behavior of	process in	search	net hunting
									birds/fish	metallurgy		
Exploration vs.	Balanced through	Strong	Balanced	Balanced	Balanced	Exploration	Balanced	Balanced	Balanced	Exploration	Exploitation-	Balanced
Exploitation	adaptive	exploration				initially, then				initially, then	focused	
	mechanisms	initially, then				exploitation				exploitation		
		exploitation										
Memory Usage	Utilizes historical	Pheromone trails	Historical positions	Habitat	Best solutions	Population-	Pack leader	Harmony memory	Historical best	Simulated states	Tabu list	Whale
	data for	as indirect	of bats	suitability index	pollinate	based, uses	memory		positions			positions
	perturbation	memory				historical data						memory
Main Operators	Local search,	Pheromone	Echolocation,	Migration,	Global and	Selection,	Encircling prey,	Pitch adjustment,	Velocity and	Temperature-	Tabu list and	Encircling prey,
	crossover,	update, path	frequency tuning	mutation	local	crossover,	attacking,	random selection	position update	based state	neighborhood	bubble-net
	mutation,	selection			pollination	mutation	searching			changes	search	hunting
	perturbation											
Adaptivity	Adaptive step size	Pheromone	Frequency	Migration rates,	Switching	Mutation and	Adaptation in	Adjustments in	Inertia weight,	Temperature and	Adaptive tabu list	Adaptive
	and perturbation	evaporation rate	adjustment,	mutation rates	probability	crossover	search phases	harmony memory	cognitive and	cooling schedule	size	hunting
	intensity		loudness and pulse			probabilities		consideration rate	social			mechanism
			rate			_			coefficients			
Convergence	Generally fast due	Moderate	Fast	Moderate	Fast	Fast	Fast	Moderate	Fast	Moderate	Slow to moderate	Fast
Speed	to adaptive											
	mechanisms											
Computational	Moderate to high,	Moderate	Moderate	Moderate	Moderate	Moderate	Moderate	Moderate	Low to moderate	Low	Moderate to high	Moderate
Complexity	depends on											
Damanatan	parameter settings		N 4 - d - we to by	N4 - days take	N A - al - u - t - lu -		N 4	N 4 - al - u - t - l - i	N 4 a d a watta h a		N 4 - d - u - t - b -	N 4
Parameter	woderately	Hignly sensitive	woderately	Woderately	Moderately	Hignly sensitive	Moderately	Nioderately	woderately	Hignly sensitive	woderately	Moderately
Sensitivity	sensitive; requires	to pheromone	sensitive	sensitive	sensitive		sensitive	sensitive	sensitive		sensitive	sensitive
Caalability	Luning	parameters	Cood	Caad	Card	Caad	Cand	Caad	Caad	Madavata	Caad	Cand
Scalability	dimonsional	woderate	6000	Guud	6000	G000	6000	Guud	6000	woderate	6000	6000
	dimensional											
El avrila il itar	problems	Madanata	N 4 a da va ta	Madavata	Madavata	Llink	Madavata	Madavata	Llink	Llink	Madavata	Madavata
FIEXIDIIILY	incorporato	iviouerate	iviouerate	wouerate	woderate	חוצוו	iviouerate	woderate	חוצנו		iviouerate	iviouerate
	various stratogies											
	various strategies								1		1	

447

449 • Exploration vs. Exploitation: The algorithm's balance between searching new areas (exploration) and refining known good

450 areas (exploitation).

- 451 Memory Usage: How the algorithm uses past information to guide future searches.
- 452 Main Operators: The primary mechanisms or processes the algorithm uses to find solutions.
- 453 Adaptivity: The algorithm's ability to adjust its parameters dynamically during the optimization process.
- 454 Convergence Speed: How quickly the algorithm typically finds a solution.
- 455 Solution Quality: The effectiveness of the algorithm in finding high-quality solutions.
- 456 Computational Complexity: The computational resources required by the algorithm, often related to time and memory usage.
- 457 Parameter Sensitivity: The degree to which the algorithm's performance is affected by its parameter settings.
- 458 Scalability: The algorithm's capability to handle problems of increasing size or complexity.
- 459 Flexibility: The algorithm's adaptability to different types of optimization problems.
- 460 Diversity Maintenance: How the algorithm ensures a diverse set of solutions to avoid premature convergence.
- 461 Typical Applications: Common fields or problems where the algorithm is frequently applied.

^{448 •} Inspiration: The natural or artificial process that inspired the algorithm's development.

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462 **3.0 Description of benchmarking algorithms, experiments, and functions**

- ⁴⁶³ This section describes the experimental examination used to benchmark FFO. For a start, FFO was
- 464 examined against 13 other commonly used optimization algorithms, namely, the Ant Colony
- ⁴⁶⁵ Optimization (ACO), Bat Algorithm (BA), Biogeography-Based Optimization (BBO), Cuckoo
- 466 Search (CS), Firefly Algorithm (FA), Flower Pollination Algorithm (FPA), Genetic Algorithm
- 467 (GA), Grey Wolf Optimizer (GWO), Harmony Search (HS), Particle Swarm Optimization (PSO),
- 468 Simulated Annealing (SA), Tabu Search (TS), and Whale Optimization Algorithm (WOA). All of
- these algorithms were used in their default settings², and a brief description of each is presented herein for completion. We invite interested readers to review the original publications for these
- herein for completion. We invite interested readers to review the origin
 algorithms to learn more about their settings and applications.

472 3.1 Ant Colony Optimization (ACO)

The Ant Colony Optimization (ACO) was formulated by Dorigo and colleagues [21] based on the 473 pheromone-laying behavior observed in certain ant species. This method uses ants as artificial 474 agents to simulate the decision-making process of real ants in selecting paths. As ants traverse 475 paths, they deposit pheromones that guide subsequent ants toward promising solutions. The ACO 476 algorithm is a probabilistic approach to problem-solving where the search space is represented as 477 a graph, and paths through this graph are evaluated based on the intensity of pheromone deposits. 478 The algorithm's efficiency hinges on several parameters: alpha (influence of pheromone on path 479 selection, set at 1.0), beta (influence of heuristic information on path choice, set at 2.0), and rho 480 (rate of pheromone evaporation, set at 0.5). This algorithms has been successfully applied to 481 network routing, scheduling, and other optimization problems that involve finding optimal paths 482 through graphs [22]. 483

inough gruphs [22].

484 3.2 Bat Algorithm (BA)

485 Developed by Yang et al. in 2012 [23], the Bat Algorithm (BA) was designed to mimic the 486 echolocation behavior of bats. This algorithm models bats that emit sound waves to navigate and 487 locate prey, translating this biological mechanism into a search and optimization strategy. In BA, 488 each simulated bat adjusts its flight based on velocity, loudness, and echolocation frequency,

- which dynamically changes from exploration to exploitation phases depending on the proximity
- 490 to optimal solutions. Key parameters of BA include alpha (initial loudness, set at 0.5), gamma (rate
- of loudness decrease and emission rate increase, set at 0.5), and frequency range (f_{min} at 0, f_{max} at
- 492 2.0). BA is adept at tackling complex problems characterized by continuous and multimodal search
- spaces and has shown effectiveness in engineering design and dynamic optimization tasks [24].

² The default setting for FFO include:

FirefighterOptimization:

def __init__(self, objective_func, dimension, num_agents=100, max_iter=500, no_improve_limit=30, bounds=(-5.12, 5.12), step_size=1.0, crossover_probability=0.5, mutation_probability=0.1, initial_temp=100.0, cooling_rate=0.95, verbose=False, use_additional_conditions=False, target_fitness=1e-5):)

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494 *3.3 Biogeography-Based Optimization (BBO)*

Introduced by Simon in 2008 [25], Biogeography-Based Optimization (BBO) leverages migration 495 concepts from biogeography to solve optimization problems. BBO operates on the premise that 496 species migrate between habitats, affecting their survival and reproduction rates. The algorithm 497 features migration operators that simulate gene flow by exchanging solution features, akin to 498 species migration in nature. Key components include the habitat suitability index, which evaluates 499 the desirability of solutions, and migration rates that determine the exchange intensity between 500 solutions. BBO also uses mutation to enhance genetic diversity and avoid premature convergence 501 on suboptimal solutions. The BBO algorithm has proven effective in network design, power 502 systems optimization, and other applications where geographic considerations are crucial [26]. 503

504 3.4 Cuckoo Search (CS)

Cuckoo Search (CS) was developed by Yang and Deb [27]. This algorithm is inspired by the 505 obligate brood parasitism of some cuckoo species by laying their eggs in the nests of other host 506 birds. If a host bird discovers the eggs are not its own, it will either throw them away or abandon 507 its nest. The algorithm uses this idea to lay a new solution (egg) into a randomly chosen nest, and 508 the best nests with high-quality eggs will be carried over to the next generations. CS is known for 509 its simplicity and flexibility. It has been effectively applied in solving problems like structural 510 design, scheduling, and routing problems where the search space is discrete, and the global 511 optimum is hidden among many local optima [28]. 512

513 3.5 Firefly Algorithm (FA)

The flashing behavior of fireflies inspires the Firefly Algorithm (FA). Such a flashing behavior 514 acts as a signal system to attract other fireflies. FA, developed by Yang in 2008 [29], uses these 515 biologically inspired techniques to handle optimization problems and functions. Fireflies in the 516 algorithm search the space by moving towards brighter and more attractive fireflies. The 517 attractiveness is proportional to the brightness, and both decrease as their distance increases. The 518 landscape of the objective function determines the brightness of a firefly. A key advantage of FA 519 is its ability to deal with multimodal optimization problems, as it naturally divides the population 520 521 into subgroups that converge to different optima. Some of the key settings in the FA algorithm include alpha (a randomness factor that affects the movement of a firefly and helps fireflies explore 522 the search space beyond the immediate neighboring fireflies, selected at 0.5), beta (controls how 523 strongly other fireflies are drawn towards it, selected at 1.0), and gamma (influences how the 524 attractiveness of a firefly decreases with distance, selected at 1.0). This feature makes it 525 particularly useful for complex functions with multiple local optima. FA has been applied to 526 problems like economic dispatch, clustering, and image processing [30]. 527

528 3.6 Flower Pollination Algorithm (FPA)

529 Devised by Yang et al. in 2012 [31], the Flower Pollination Algorithm (FPA) algorithm emulates

the natural pollination processes of flowers. It aims to optimize solutions by alternating between

self-pollination and cross-pollination mechanisms that are naturally facilitated by natural vectors

532 like insects, wind, or water. This approach maintains solution diversity and promotes effective

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533 convergence. In FPA, solutions are represented as flowers whose attractiveness—determined by 534 fitness—guides the pollination process. The algorithm uses local pollination for minor adjustments 535 within the immediate search area and global pollination, employing Levy flights for broader 536 searches to escape local optima. FPA's dual strategy has been effectively applied to engineering 537 design and economic load dispatch challenges [32].

538

539 *3.7 Genetic Algorithm (GA)*

Holland [33] formulated the Genetic Algorithm (GA) as a computational analog to natural 540 selection, embodying the principle of survival of the fittest. GA begins with a population of 541 randomly generated individuals, evolving over generations to optimize solutions. Selection is 542 fitness-based, favoring solutions that perform better under a defined fitness function. GA 543 incorporates mutation and crossover as genetic operators to introduce variability and new traits 544 into offspring. Typical parameters include a population size of 100, a mutation rate of 0.1, and a 545 crossover rate of 0.1. This algorithm is widely utilized across fields such as optimization, automatic 546 programming, and machine learning, where it helps solve complex problems efficiently [34]. 547

548 3.8 Grey Wolf Optimizer (GWO)

The Grey Wolf Optimizer (GWO), introduced by Mirjalili et al. in 2014 [35], is a nature-inspired 549 metaheuristic algorithm inspired by grey wolves' social structure and hunting behavior. Grey 550 wolves exhibit a distinct hierarchical system consisting of alpha, beta, delta, and omega wolves, 551 with each tier playing a specific role within the pack. In GWO, this hierarchy is mirrored in the 552 solution process: the alpha wolf represents the optimal solution, followed by beta and delta as the 553 second and third best solutions, respectively, while omega wolves embody the remaining candidate 554 solutions. The algorithm leverages this structure to simulate the wolves' hunting strategy, which is 555 segmented into three phases: tracking, encircling, and attacking prey, each reflecting a critical 556 phase of the optimization process. GWO is adept at navigating complex, multidimensional 557 landscapes, making it valuable in fields such as mechanical engineering design and renewable 558 energy optimization, where the search spaces often exhibit high nonlinearity and multimodality. 559

560 3.9 Harmony Search (HS)

Developed by Geem et al. in 2001 [36], Harmony Search (HS) is an optimization algorithm 561 inspired by the improvisational process of musicians tuning their instruments to achieve aesthetic 562 harmony. This algorithm iteratively adjusts solution vectors in a similar fashion to musicians' 563 adjust pitches to optimize a given function. HS employs a stochastic approach rather than a 564 gradient-based method, enhancing its efficacy in addressing non-differential and discrete 565 problems. The algorithm's performance is governed by two primary parameters: the harmony 566 memory consideration rate (set at 0.9), which dictates the likelihood of selecting existing memory 567 solutions for new harmonies, and the pitch adjustment rate (set at 0.3), which determines how 568 much the chosen solutions are modified. HS has shown significant utility in solving complex 569 570 engineering problems such as structural and water network design, where traditional methods may 571 struggle due to the extensive search spaces involved.

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572 *3.10 Particle Swarm Optimization (PSO)*

Particle Swarm Optimization (PSO) was introduced by Kennedy and Eberhart in 1995 [37]. This 573 algorithm simulates the social behaviors observed in flocks of birds or schools of fish. This 574 metaheuristic optimizes problem solutions by iteratively enhancing a population of candidate 575 solutions based on the personal and collective experiences of the particles. For example, the PSO 576 starts with randomly initialized particles (solutions) and updates their positions within the search 577 space by balancing personal best achievements and global knowledge shared across the swarm. 578 The algorithm is known for its simplicity and adaptability, often requiring few parameter 579 adjustments. It utilizes three key parameters: swarm size (typically 100 particles), cognitive 580 coefficient (influence of the particle's own memory, set at 1.0), and social coefficient (influence 581 of neighboring particles, also set at 1.0). These factors influence the dynamics of particle 582 movements towards optimal solutions. PSO is particularly effective in continuous, high-583 dimensional environments and has been applied successfully across various domains, including 584 electrical power systems, robotics, and bioinformatics [38]. 585

586 3.11 Simulated Annealing (SA)

Simulated Annealing (SA) was developed by Kirkpatrick et al. in 1983 and Cerny in 1985 [39]. 587 This is a probabilistic method designed to approximate the global optimum of a function. SA is 588 inspired by the metallurgical process of annealing, where materials are heated and then gradually 589 cooled to improve their structural properties. This process is mimicked by allowing a system to 590 explore higher energy states (solutions) by heating, thereby overcoming local optima, followed by 591 slow cooling to stabilize at a lower energy state (optimal solution). The algorithm makes random 592 transitions to neighboring solutions, accepting improvements outright and worse solutions based 593 594 on a decreasing probability over time. Key parameters include the initial temperature (set at 100) and cooling rate (set at 0.95). SA has been successfully applied across various domains, from 595 economics to computational science [40]. 596

597 *3.12 Tabu Search (TS)*

Tabu Search (TS) was introduced by Glover in the late 1980s [41] as a metaheuristic that extends 598 599 beyond local search methods. TS extends such methods by adopting a memory structure, the tabu list, to avoid cycling back to previously encountered suboptimal solutions. This list temporarily 600 bans certain moves, helping the algorithm to escape local optima and explore less favorable 601 solutions that might lead to a globally optimal solution. TS is adaptable, allowing periodic resetting 602 of the tabu status to balance exploration and exploitation. It is particularly effective in complex 603 scheduling, logistical planning, and assignment problems, where its ability to navigate challenging 604 solution spaces is applications [42]. 605

606 3.13 Whale Optimization Algorithm (WOA)

- ⁶⁰⁷ The Whale Optimization Algorithm (WOA) was developed by Mirjalili and Lewis in 2016 [43].
- ⁶⁰⁸ The WOA draws inspiration from the bubble-net feeding behavior of humpback whales. This
- algorithm simulates the whales' strategies of encircling prey and using a spiral path to close in,
- 610 which are mirrored in the shrinking encircling mechanism and spiral updating position phases of

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- the algorithm. These methods allow the WOA to balance exploration and exploitation
- dynamically, making it adept at handling complex, non-separable, and nonlinear optimization
- 613 problems. WOA's effectiveness is demonstrated in its applications across mechanical design and
- 614 industrial engineering, where it optimizes a variety of challenging problem landscapes [44].

615 **4.0 Description of utilized benchmarking functions**

⁶¹⁶ This section describes 24 benchmark functions commonly used in benchmarking analysis.

617 4.1 Ackley Function

The Ackley function has a two-dimensional form with a relatively uniform plane [45]. This

function also has several dozen local minimums and one global extreme of significantly smaller

value than most of the local minimums. This function allows very efficient testing of optimization

algorithms as regards stopping at local extremes. The Ackley function is designed to test the ability

- of optimization algorithms to escape local minima and converge towards a global minimum in a complex landscape. The global minimum is at x=0 where f(x)=0. This function has the following
- 625 complex failescape. The global minimum is at x=0 where f(x)=0. This function has the following 624 form:

625
$$f(x) = -20 \exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$$
 Eq. 1

626 4.2 Alpine Function

The Alpine function is a multimodal and non-smooth function and hence can provide significant challenges in terms of local minima and ruggedness tests [46]. This function examines the ability of optimization algorithms to handle non-differentiable points with abrupt changes. The global minimum for this function occurs at x=0 where f(x)=0. This function can be useful for testing optimization algorithms in real-world problems involving non-smooth dynamics, such as mechanical systems with friction or other resistive forces. This function has the following form:

633
$$f(x) = \sum_{i=1}^{n} |x_i \sin(x_i) + 0.1x_i|$$
 Eq. 2

634 4.3 Booth's Function

This function presents a simple test case for algorithm testing with a convex with a single global minimum at (x,y) = (1,3) where f(x,y) = 0. This function has the following form:

637
$$f(x,y) = (x+2y-7)^2 + (2x+y-5)^2$$
 Eq. 3

638 4.4 Cross-in-Tray Function

- 639 This function is known for its challenging landscape, characterized by a high degree of
- multimodality [47]. The function contains several deep holes, indicative of global minima, which
- are located symmetrically in the function's domain, and may present a significant challenge in the
- 642 convergence process of algorithms to navigate complex landscapes and avoid local minima in
- 643 favor of locating and confirming global minima. The global minima occur at approximately
- $644 \quad (x,y) = (1.34941, -1.34941), (-1.34941, 1.34941), (1.34941, 1.34941), (-1.34941, -1.34941) \quad \text{with}$
- 645 $f(x,y) \approx -2.06261$. This function has the following form:

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646
$$f(x,y) = -0.0001 \left(\left| \sin(x) \sin(y) \exp\left(\left| 100 - \frac{\sqrt{x^2 + y^2}}{\pi} \right| \right) \right| + 1 \right)^{0.1}$$
 Eq. 4

647 4.5 Drop-Wave Function

This function features a rippled wave surface that turn challenging to optimize due to its frequent local minima and a pronounced global minimum [48]. This is a multimodal function with a global minimum at (x,y)=(0,0) where f(x,y)=-1. It tests an algorithm's capability to navigate through frequent oscillations to find the lowest point and is particularly relevant for simulations and optimizations in fields involving vibrational analysis and wave propagation (i.e., acoustics and materials science, etc.). This function has the following form:

654
$$f(x,y) = -\frac{1+\cos(12\sqrt{x^2+y^2})}{0.5(x^2+y^2)+2}$$
 Eq. 5

655 4.6 Easom Function

⁶⁵⁶ The Easom function is a highly unimodal benchmark function stemming from its narrow global

657 peak that is surrounded by a flat landscape [49]. This function has a constant plane over the vast

majority of the domain with one global minimum, at $(x,y)=(\pi,\pi)$ where f(x,y)=-1, that is difficult

to locate due to the flatness of the surrounding area. Oftentimes, this function is used in testing the

660 precision and convergence characteristics of optimization algorithms, and their ability to hone in

on and precisely converge to a sharply defined minima. This function has the following form:

662
$$f(x, y) = -\cos(x)\cos(y)\exp(-((x - \pi)^2 + (y - \pi)^2))$$
 Eq. 6

663 4.7 Eggholder Function

The Eggholder function has a highly irregular and complex surface characterized by an uneven plane with several local minimums (of values like its only global minimum) [50,51]. The global minimum is found at (x,y)=(512,404.2319) where $f(x,y)\approx-959.6407$. This function is frequently used in benchmarking sophisticated global optimization algorithms, especially those intended to solve rugged and unpredictable landscape-based problems. This function has the following form:

669
$$f(x,y) = -(y+47)\sin\left(\sqrt{\left|\left(\frac{y+x}{2+47}\right)\right|}\right) - x\sin\left(\sqrt{|x-(y+47)|}\right)$$
 Eq.7

670 4.8 Expanded Schaffer's F6 Function

This is an expansion of the original Schaffer's function that hopes to test for algorithm effectiveness over a broader area with a more complex landscape. More specifically, the function is highly multimodal and oscillatory and hence presents a significant challenge in identifying the global minimum amidst numerous local minima. The function has a global minimum at (x,y)=(0,0) where f(x,y)=0. This function can be used in testing spatial algorithms that may be applied in fields like geographic information systems and molecular dynamics where spatial relationships and dynamics are crucial.

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678
$$f(x) = 0.5 + \frac{\sin^2(\sqrt{x^2 + y^2}) - 0.5}{[1 + 0.001(x^2 + y^2)]^2}$$
 Eq. 8

679 4.9 Expanded Zakharov Function

This is an extension of the Zakharov function [46] and provides a more challenging scenario for testing optimization algorithms by combining linear, quadratic, and quartic terms. This function has a single global minimum at x=0 where f(x)=0. This function is used to evaluate the performance of large-scale optimization algorithms in areas such as financial modeling and energy systems, where complex interactions between variables are common.

685
$$f(x) = \sum_{i=1}^{n} x_i^2 + (\sum_{i=1}^{n} 50ix_i)^2 + (\sum_{i=1}^{n} 50ix_i)^4$$
 Eq. 9

686 4.10 Goldstein-Price Function

This function was created by Goldstein and Price [52] to provide a multimodal complex landscape with sharp peaks and valleys. The same function has several local minima and a global minimum found at (x,y) = (0,-1) with f(x,y) = 3. This function has the following form:

690
$$f(x,y) = [1 + (x + y + 1)^2(19 - 14x + 3x^2 - 14y + 6xy + 3y^2)] \times [30 + 2(sx - 3y)^2(18 - 32x + 12x^2 + 48y - 36xy + 27y^2)]$$
 Eq. 10

692 4.11 Griewank Function

The Griewank function has large, flat areas interrupted by periodic narrow, deep valleys [53]. This function is highly multimodal and oscillatory and can challenge the algorithm's ability to find the global minimum (at x=0) amidst frequent changes in gradient. It is particularly used to test the efficiency of algorithms in handling complex oscillations and multimodal functions with application within acoustic waveguides design and structural engineering with regard to vibrations. This function has the following form:

699
$$f(x) = 1 + \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod_{i=1}^{n} \cos(x_i/\sqrt{i})$$
 Eq. 11

700 4.12 Himmelblau's Function

Developed by Himmelblau [54], this function is characterized by multiple global minima, which
 makes it interesting for testing the robustness of optimization algorithms to locate and distinguish
 between multiple optima within a complex landscape. This function has four identical global
 minima located at

705
$$(x,y)=(3,2),(-2.805118,3.131312),(-3.779310,-3.283186),(3.584428,-1.848126)$$
 where

706 f(x,y)=0. This function has the following form:

707
$$f(x,y) = (x^2 + y - 11)^2 + (x + y^2 - 7)^2$$
 Eq. 12

- 708 4.13 Holder Table Function
- 709 The Holder Table function features several deep and narrow global minima and is designed to
- challenge optimization algorithms in finding and recognizing global solutions in a multimodal
- ⁷¹¹ space [55]. The function's global minima are symmetrically located around the origin, with four

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- known global minima where f(x,y) = -19.2085. This function can serve as a good evaluation metric
- 713 for multimodal optimization capabilities in logistics and routing problems where multiple
- ria equivalent optimal routes need to be evaluated. This function has the following form:

715
$$f(x,y) = -\left|\sin(x)\cos(y)\exp(\left|1 - \frac{\sqrt{x^2 + y^2}}{\pi}\right|)\right|$$
 Eq. 13

716 4.14 Levy Function N.13

This is a variant to the Levi function. This variant is designed to test algorithms against steep gradients and local optima. More specifically, this function has steep ridges and a complex global structure with a global minimum at (x,y)=(1,1) where f(x,y)=0. The Levi can be useful in examining algorithms that need to handle sudden changes in gradient effectively. This function has the following form:

722
$$f(x,y) = \sin^2(3\pi x) + (x-1)^2(1+\sin^2(3\pi y)) + (y-1)^2(1+\sin^2(2\pi y))$$
 Eq. 14

- 723 4.15 Matyas Function
- This function was created by Matyas [56] as convex function that could serve as an elementary
- test case for basic functionality and efficiency of optimization algorithms in a controlled setting.
- The Matyas function has a global minimum at (x,y) = (0,0) where f(x,y)=0. This function has the
- 727 following form:

728
$$f(x,y) = 0.26(x^2 + y^2) - 0.48xy$$
 Eq. 15

729 4.16 Michalewicz Function

This function was created by Michalewicz [57] to be especially difficult for evolutionary algorithms to solve. This function is highly multimodal, with sharp peaks and valleys that are sensitive to the variable m, which controls the steepness of the valleys and ridges. The function's global minimum becomes more difficult to locate as m increases (with a typical value of m=10). This function can be used in the testing and development of genetic and evolutionary algorithms, particularly effective for applications requiring high precision in aerodynamics and biomechanical engineering. This function has the following form:

737
$$f(x) = -\sum_{i=1}^{n} \sin(x_i) \sin^{2m}(\frac{ix_i^2}{\pi})$$
 Eq. 16

738 4.17 Rastrigin Function

- This function is named after Rastrigin [58] and presents an example of a highly non-linear multimodal function with frequent local minima. The global minimum is at x=0 where f(x)=0.
- 741 The function is particularly designed to test the algorithm's capability to escape local minima and
- ⁷⁴² is widely used in the testing and development of algorithms in evolutionary computation and real-
- vorld scenarios where noise and local minima are prevalent, such as in electronic circuit design.

744 This function has the following form:

745
$$f(x) = 10n + \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i)]$$
 Eq. 17

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746 4.18 Rosenbrock Function

- This function was designed by Rosenbrock in 1960 as a non-linear and non-convex function to
- test the performance of optimization algorithms over rugged terrain with a narrow, curved valley
- leading to a global minimum [59]. This function has a global minimum inside a long, narrow,
 parabolic shaped flat valley. In general, finding the valley is straightforward but converging to the
- 751 global minimum is difficult. A common multidimensional generalization of this function has the
- 752 following form:

753
$$f(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2]$$
 Eq. 18

754 4.19 Schaffer Function N. 2

This function is a variant belonging to the family of functions introduced by Schaffer, which are used to evaluate the performance of optimization algorithms in handling oscillating landscapes with narrow valleys [60]. The function is non-convex and multimodal, with a global minimum at (x,y)=(0,0) where f(x,y)=0. This variant can be sensitive to initial conditions due to the presence

of sharp peaks and deep valleys. This function has the following form:

760
$$f(x,y) = 0.5 + \frac{\sin^2(x^2 - y^2) - 0.5}{[1 + 0.001(x^2 + y^2)]^2}$$
 Eq. 19

761 4.20 Schwefel Function

The Schwefel function is a classic optimization test problem introduced by Schwefel [61]. This function is initially created for evolutionary algorithms and has complex and non-linear large number of local minima (with a global minimum located near the bounds of the search space at x=(420.9687,...,420.9687) where $f(x)\approx 0$). This function can be helpful in in fields such as aerospace for optimizing the shapes and trajectories of dynamic flying bodies. This function has the following form:

768
$$f(x) = 418.9829n - x_i \sin(\sqrt{|x_i|})$$
 Eq. 20

769 4.21 Sphere Function

The Sphere function is one of the classical and simplest benchmark functions often used to test preliminary optimization algorithms in terms of convergence and accuracy. The Sphere function is continuous, convex, and unimodal with a global minimum at $x^* = 0$ where $f(x^*) = 0$. This function has the following form:

774
$$f(x) = \sum_{i=1}^{n} x_i^2$$
 Eq. 21

775 4.22 Styblinski-Tang Function

The Styblinski-Tang function has steep valleys and multiple local minima and hence is recognized

for its utility in testing optimization algorithms [62]. This function is multimodal, with each

variable contributing quadratically and quartically to the output. This function can be suitable for

- evaluating the efficiency of algorithms in high-dimensional spaces and their capability to scale
- with increasing dimensionality (which makes it appropriate for practical engineering problems

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- involving material design and circuit optimization with multiple variables required to be 781
- simultaneously optimized). The global minimum for this function is at $x_i = -2.903534x$ (for all *i*) 782 where f(x) = -39.16599n (where n is the number of dimensions). This function has the following 783
- form: 784

785
$$f(x) = \frac{1}{2} \sum_{i=1}^{n} (x_i^4 - 16x_i^2 + x_i 5)$$
 Eq. 22

4.23 Three-hump Camel Function 786

This function is named for its shape, resembling three camel humps and is designed as a simple 787 and effective benchmark for testing optimization algorithms [63]. This function can be used to 788 evaluate an algorithm's capability to escape local minima and find the global minimum. The 789 Three-hump Camel function has a global minimum at (x,y)=(0,0) where f(x,y)=0. This function 790 has the following form: 791

792
$$f(x,y) = 2x^2 - 1.05x^4 + \frac{x^6}{6} + xy + y^2$$
 Eq. 23

793 4.24 Whitley's Function

794 Whitley's function is known for its complex landscape with a high number of local minima [51]. This function is tests algorithms for their ability to distinguish subtle gradient changes and avoid 795 premature convergence. This function can be used for complex, real-world problems such as 796 landscape exploration and molecular configuration. This function has the following form: 797

798
$$f(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{100(x_i^2 - x_j)^2 + (1 - x_j)^2}{4000} - \cos\left(200(x_i^2 - x_j)^2 + (1 - x_j)^2\right) + 1 \right)$$
 Eq. 24

Table 2 includes the name of each function, its typical domain range, primary characteristics, 799

known challenges, and the dimensionality for which the function is typically evaluated. Figure 1 800 compares these functions visually.

801

	Function Name	Domain Range	Characteristics	Challenges	Typical Dimensionality	Minima/ Minimum
	Ackley	(-5, 5)	Nearly flat outer region, large hole	Global minimum difficult to find due to flatness	Multiple	0
-	Alpine	(-10, 10)	Multimodal, peaks and valleys	Peaks make locating global minima challenging	Multiple	0
	Beale's	(-4.5 <i>,</i> 4.5)	Multimodal	Several local minima	2	0
_	Booth's	(-10, 10)	Smooth, few local minima	Simplicity, limited challenge	2	0
	Cross-in-Tray	(-10, 10)	Highly multimodal	Multiple global minima spread out	2	-2.0626
	Drop-Wave	(-5.12, 5.12)	Central peak surrounded by ring of minima	Central peak difficult to stabilize on	Multiple	-1

Table 2 Holistic comparison of the examined functions. 802

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Easom	(-100, 100)	Narrow peak, unimodal	Extremely narrow attraction region	2	-1
Eggholder	(-512, 512)	Large search space, numerous local minima	Complex landscape, many minima	Multiple	-959.640
Expanded Schaffer's F6	(-10, 10)	Numerous local minima, complex landscape	Maintaining algorithm stability	2	0
Expanded Zakharov	(-10, 10)	Combines parabolic and linear terms	Optimization of combined effects	Multiple	0
Goldstein- Price	(-2, 2)	Complex topology, multimodal	Local minima near boundaries	2	3
Griewank	(-600, 600)	Many widespread minima	Large search space, many local minima	Multiple	0
Himmelblau's	(-5 <i>,</i> 5)	Multiple global minima	Identifying correct global minimum	2	0
Holder Table	(-10, 10)	Symmetric, multiple global minima	Symmetry can confuse algorithms	Multiple	-19.2085
Levy N.13	(-10, 10)	Steep valleys, complex structure	Pronounced ridges and steep drops	2	0
Matyas	(-10, 10)	Smooth, unimodal	Limited complexity	2	0
Michalewicz	(0, π)	Designed for evolutionary algorithms	Sharp, narrow valleys	Multiple	Multiple
Rastrigin	(-5.12, 5.12)	Highly multimodal, oscillating	Numerous local minima, large search space	Multiple	0
Rosenbrock	(-2.048, 2.048)	Non-convex, narrow curved valley	Finding the global minimum in the narrow valley	Multiple	0
Salomon's	(-100, 100)	Strong global structure, multimodal	Multiple deceptive local minima	Multiple	0
Schaffer N. 2	(-100, 100)	Sharp ridges, flat areas	Balancing exploration and exploitation	Multiple	0
Schwefel	(-500, 500)	Sinusoidal, maxima and minima	Identifying global minimum amid deceptive maxima	Multiple	0
Sphere	(-5.12, 5.12)	Smooth, convex, unimodal	Simplicity may not challenge advanced algorithms	Multiple	0
Styblinski- Tang	(-5, 5)	Steep valleys, multimodal	Harsh penalties for incorrect solutions	Multiple	-39.165 <i>n</i>
Three-hump Camel	(-5, 5)	Three humps, unimodal	Misleading local minima	2	0
Whitley's	(-10, 10)	Highly complex and multimodal	Complexity and size of search space	Multiple	0

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804

805

Fig. 1 Visual representation of the utilized test and benchmark functions

806 **5.0 Discussion, experiments, results, and analysis**

Two sets of experiments were conducted to evaluate the performance of the FFO algorithm. In the first, we examine the performance of the FFO algorithm and the other listed algorithms against the aforenoted benchmark functions in 2D settings. Then, we examine the algorithmic performance against the scalable functions at higher dimensions (20D and 50D). In all cases, the experiments entitled a comparison of algorithmic performance in terms of best fitness and fitness history

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achieved, time taken to execute the analysis per algorithm, trajectory analysis, and a combined combination of metrics. All of these items are discussed herein in detail.

814 The best fitness is defined as the most favorable value of the objective function obtained in an analysis by an algorithm across all of its iterations or agents. This metric is traced progressively, 815 with the algorithm updating this metric whenever an improved solution is found. Then, the 816 exaction time is defined as the total time from the start of its execution until it terminates. 817 Trajectory refers to the sequence of points that document the position of agents (or the best agent) 818 in the search space. This distance is calculated by summing up the Euclidean distances between 819 consecutive points in the trajectory. Further, four additional categories were used to evaluate the 820 selected algorithms. These include documenting the performance of algorithms in terms of 821 identifying the functions that took the longest (and shortest) time to solve and those that achieved 822 the most (and least) accuracy. 823

To facilitate a leaner comparison, the execution time and distance explored metrics were combined into a new metric named the Distance per Unit Time metric. This metric directly measures the average distance covered per unit of time, which is directly interpretable as:

827 Distance per Unit Time metric $= \frac{Total \ distance}{Excution \ time}$

This measures how much distance is covered on average per second. Therefore, higher values indicate more efficient exploration of the search space.

The conducted analysis was ran and evaluated in a Python 3.10.5 environment using an Intel(R) Core(TM) i7-9700F CPU @ 3.00GHz and an installed RAM of 32.0GB. To ensure fairness, the control parameters of FFO and the 13 metaheuristics employed in the performance evaluation simulation were presented earlier and are found in our simulation script. In all cases, all algorithms ran for 100, 1000, and 3000 iterations and with 10, 50, and 100 agents. As mentioned above, the first leg of the analysis focused on all benchmarking functions at 2D, and the second leg focused on scalable functions (12 out of the original 24 functions) at higher dimensions (20D and 50D).

837 2D setting

Table 3 and Fig. 2 list the overall obtained results from the analysis carried out on all algorithms 838 and functions in 2D setting. This table presents a comparative performance analysis of various 839 optimization algorithms based on metrics such as best fitness, execution time, and distance 840 metrics. Ant Colony Optimization, for instance, shows a mean best fitness of 1.22E+05 with high 841 variability (standard deviation of 9.48E+05) and ranges from -563 to 7.35 in its best fitness 842 performance. In terms of spatial efficiency, it maintains a low average distance metric (mean of 843 0.092). On the other hand, the Firefly algorithm demonstrates a broader range in execution time, 844 peaking at 1803 seconds, and exhibits a significantly wider spread in the distance metric, reaching 845

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- ⁸⁴⁶ up to 91,600. The FFO with additional conditions³ "OFF" significantly improves best fitness and
- execution efficiency and large exploration capabilities to a maximum of 1.20E+08. It is quite clear
- that the FFO ranked well in all metrics and achieved the top ranking in terms of the Distance per
- 849 Unit Time metric.

850



Fig. 2 Ranking of algorithms in 2D settings

³ The FFO (with additional conditions OFF) runs the FFO in a similar fashion to the other algorithms (i.e., to the same number of iterations without any additional stopping criteria). The counterpart version (with additional conditions ON) runs the FFO with all stopping criteria as described above. A true comparison between all algorithms should rely on the OFF version and hence is maintained herein.

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Table 3 Overall results for 2D setting

		Best	itness			Executio	n Time (s)			Distanc	e metric		Distance
Algorithm	mean	std	min	max	mean	std	min	max	mean	std	min	max	Time
Ant Colony Optimization	1.22E+05	9.48E+05	-5.63E+02	7.35E+00	7.35E+00	9.39E+00	9.00E-02	3.51E+01	9.26E-02	1.36E+00	0.00E+00	2.00E+01	1.26E-02
Bat Algorithm	-1.83E+00	4.58E+02	-9.77E+02	2.63E+00	2.63E+00	3.67E+00	1.88E-02	1.99E+01	3.15E+01	1.58E+02	0.00E+00	1.14E+03	1.20E+01
Biogeography- Based Optimization	-9.72E+00	1.93E+02	-8.81E+02	7.34E+00	7.34E+00	9.76E+00	7.13E-02	4.42E+01	2.64E+03	1.17E+04	0.00E+00	1.41E+05	3.60E+02
Cuckoo Search	-4.46E+01	1.92E+02	-9.60E+02	5.99E+01	5.99E+01	1.18E+02	5.19E-02	7.51E+02	8.48E+01	3.83E+02	0.00E+00	3.74E+03	1.42E+00
FFO (additional conditions OFF)	-4.03E+01	1.84E+02	-9.60E+02	4.61E+00	4.61E+00	7.82E+00	2.50E-02	5.23E+01	2.64E+06	1.21E+07	1.47E+03	1.20E+08	5.74E+05
FFO (additional conditions ON)	-2.04E+01	1.20E+02	-8.18E+02	9.11E-02	9.11E-02	1.25E-01	0.00E+00	7.27E-01	1.02E+05	3.86E+05	0.00E+00	3.36E+06	1.12E+06
Firefly Algorithm	-2.25E+01	1.71E+02	-8.75E+02	1.73E+02	1.73E+02	3.09E+02	1.31E-01	1.80E+03	7.31E+02	6.83E+03	0.00E+00	9.16E+04	4.23E+00
Flower Pollination Algorithm	-3.85E+01	1.75E+02	-9.41E+02	2.86E+00	2.86E+00	3.93E+00	2.33E-02	2.07E+01	6.87E+01	3.08E+02	0.00E+00	2.39E+03	2.40E+01
Genetic Algorithm	-3.02E+01	1.65E+02	-9.56E+02	1.92E+00	1.92E+00	2.48E+00	2.40E-02	1.17E+01	5.62E+00	1.44E+01	3.62E-03	1.14E+02	2.93E+00
Grey Wolf Optimizer	-1.51E+01	1.78E+02	-9.60E+02	5.38E+00	5.38E+00	7.49E+00	4.14E-02	4.01E+01	3.59E+03	1.62E+04	8.90E-02	1.44E+05	6.67E+02
Harmony Search	-3.95E+01	1.78E+02	-9.60E+02	7.52E-01	7.52E-01	1.19E+00	6.00E-03	7.33E+00	1.18E+02	4.52E+02	0.00E+00	3.42E+03	1.57E+02
Particle Swarm Optimization	-3.76E+01	1.74E+02	-9.60E+02	2.39E+00	2.39E+00	3.13E+00	2.46E-02	1.43E+01	1.07E+05	9.96E+05	3.74E+01	1.44E+07	4.45E+04
Simulated Annealing	-3.80E+00	1.34E+02	-7.18E+02	3.65E-02	3.65E-02	3.41E-02	9.97E-04	1.25E-01	1.04E+02	9.39E+01	2.30E+00	7.66E+02	2.86E+03
Tabu Search	1.02E+01	1.81E+02	-7.87E+02	8.23E-02	8.23E-02	7.41E-02	3.99E-03	2.63E-01	5.77E+02	5.14E+02	2.89E+01	1.51E+03	7.01E+03
Whale Optimization Algorithm	-4.14E+01	1.78E+02	-9.58E+02	2.45E+00	2.45E+00	3.22E+00	2.47E-02	1.47E+01	3.73E+03	1.30E+04	2.94E-02	9.68E+04	1.52E+03

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Now, we gain further insights into the performance of all algorithms across different settings 854 through the illustration presented in Fig. 3. This figure shows a sample of trends collected on the 855 selected functions to show how the algorithms perform. It is quite clear that the FFO performs 856 similarly to other algorithms. For example, the first sub-figure highlights performance variations 857 of several optimization algorithms on the Ackley function at 2D, with a specific focus on the 858 number of agents employed. The FFO with additional conditions "OFF" consistently maintains 859 lower fitness values across all agent configurations compared to most algorithms, suggesting 860 higher efficiency. This is in contrast to algorithms like Bat algorithm and Ant Colony Optimization, 861 which show a marked increase in average best fitness. The FFO with additional conditions "OFF" 862 outperforms the latter at higher agent counts, which indicates better scalability with increasing 863 agents. Similar observations can also be made in terms of the algorithmic performance with the 864 number of iterations - especially in the case of the Egg Holder function, which shows superior 865 performance for the FFO algorithm. 866



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Fig. 3 Sample of trends across different experimental settings

Here, we analyze and rank the performance of optimization algorithms across various settings, 872 specifically focusing on the dimensions, maximum iterations, and the number of agents involved 873 in terms of identifying those that took the longest (and shortest) time to solve, and that achieved 874 the most (and least) accuracy. This process was conducted at each unique setting. First, we rank 875 the algorithms by returning the top three entries of the specified metric and setting. Then, we 876 aggregate these frequencies both locally (for each setting) and globally (across all settings) to 877 provide a comprehensive view of which algorithms consistently perform well or underperform 878 across varied configurations. 879

We report that the Firefly algorithm appears predominantly in the longest to solve category with a 880 total of 27 out of 27 occurrences. This suggests that the Firefly algorithm, despite its potential 881 advantages in exploring complex landscapes, tends to have longer execution times compared to 882 other algorithms in the study. This could be due to its inherent characteristics, such as the 883 attractiveness parameter and light intensity, which might cause slower convergence, especially in 884 scenarios involving complicated objective functions. The FFO (additional conditions ON) 885 algorithm consistently appears as the fastest solver, also with 27 instances. This algorithm was 886 followed by the Tabu Search. The success in this category indicates the algorithmic potential for 887 applications requiring quick solutions where computational resources can be limited. 888

In the most accurate category, the Cuckoo Search leads with 9 occurrences, followed by the FFO (additional conditions OFF) with 5 occurnaces and various other algorithms. The notable performance of Cuckoo Search could be attributed to its unique search capabilities, leveraging Lévy flights for global search combined with a probabilistic switch to local search. On the other

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hand, the Ant Colony Optimization is predominantly featured in the least accurate category with
 instances. This might indicate ACO's challenges in maintaining high accuracy across the
 settings tested, potentially due to its reliance on pheromone trails, which might lead to premature
 convergence or difficulty in escaping local optima in certain types of problems.

Other sets of comparisons between selected algorithms can be seen in Fig. 4. For example, Fig. 4a 897 compares the average history fitness across all functions and for all algorithms. This figure shows 898 that the FFO (additional conditions OFF) demonstrates a rapid convergence initially compared to 899 others like the Bat and Grey Wolf Optimizer, which exhibit more gradual improvements. Figure 4b 900 compares the best fitness achieved in FFO, Tabu Search, the Bat, and the Grey Wolf Optimizer 901 algorithms. In this figure, FFO (additional conditions OFF) excels in deeply multi-modal landscapes 902 like the Ackley and Griewank functions. This suggests that FFO (additional conditions OFF) is 903 particularly adept at managing and escaping local optima in complex search spaces. The graph 904 also shows minimal variance in performance across different configurations (agent counts and 905 iteration limits), indicating the robustness and effectiveness of this algorithm. 906

Figure 4c shows the performance of all algorithms in tackling continuous and non-continuous functions to provide a clear comparison of how each algorithm handles different types of function landscapes. Here, the FFO (additional conditions OFF) shows comparable performance in both

states of states

the top performers, closely competing with algorithms like Particle Swarm Optimization and

912 Genetic Algorithm. The FFO (additional conditions OFF) also shows similar performance in non-

ontinuous functions. These figures further show the comparative performance and consistency of

the newly proposed FFO algorithm against those well established methods.

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Function

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920 921

919

Fig. 4 Further cross examination between the FFO and other notable algorithms

To complement the above analysis, the Friedman and Wilcoxon non-parametric statistical tests for ranking the above algorithm were carried out [64,65]. These statistical methods can evaluate and

⁹²⁴ compare the performance of optimization algorithms across multiple functions. The Friedman test

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- ranks algorithms based on their mean performance across all tested functions, where a lower rank
- ⁹²⁶ indicates superior performance (i.e., achieving better, usually lower, fitness values). Each function
- ⁹²⁷ is treated equally, and the ranks are averaged to provide an overall ranking for each algorithm. The
- ⁹²⁸ test then checks if the observed rankings are statistically significant.

On the other hand, the Wilcoxon test focuses on pairwise comparisons between algorithms. This test calculates the signed rank of the differences in performance between each pair to assess whether one algorithm consistently outperforms the other. While both tests aim to highlight the best-performing algorithms, they approach the evaluation from slightly different perspectives— Friedman emphasizes overall ranking across all scenarios, while Wilcoxon emphasizes consistency in pairwise dominance. Figure 5 shows the outcome of these tests. As one can see, the

FFO ranks 4 and 5 in these tests, respectively. PSO, GWO, and FPO rank in the top three spots.





Fig. 5 Friedman and Wilcoxon non-parametric statistical tests for 2D

938 *20D and 50D setting*

939 Similar to the previous analysis and discussion, Table 4 and Fig. 6 list the overall obtained results

⁹⁴⁰ from the analysis carried out on all algorithms and functions in higher dimensions of 20D and 50D.

⁹⁴¹ Thus, only scalable functions were used in this examination. These functions include Ackley,

942 Eggholder, Easom, Expanded Shaffer's F6, Expanded Zakharov, Griewank, Gldstien-Price,

943 Rosenbrock, Schaffer N.02, Schwefel, Sphere, and Whitely.

⁹⁴⁴ In this comparative analysis, the FFO (additional conditions OFF) demonstrates a creditable mean

best fitness of 2.88E+03, which, while not the lowest in our dataset, offers a viable trade-off

between fitness achievement and computational resources when compared to algorithms like the

947 Simulated Annealing, which has a lower mean best fitness of 5.17E-02 but at a significantly

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- reduced complexity. In terms of execution time, the FFO (additional conditions OFF) records a
- mean of 9.35E+02 seconds, positioning it as a middle-range performer. It is notably faster than
- 950 high-accuracy contenders such as the Firefly algorithm and the Biogeography-Based Optimization,
- which clocks in at mean times of 2.93E+02 seconds and 1.06E+01 seconds, respectively, reflecting
- ⁹⁵² a more efficient performance considering the relatively lower fitness figures.
- 953 On a more positive note, the FFO (additional conditions OFF) has significat exploration capability,
- as measured by the Total Distance metric, where it posts a mean of 1.58E+07. This is significantly
- higher than that of the Particle Swarm Optimization and Grey Wolf Optimizer, which stand at
- 2.98E+05 and 3.10E+04, respectively. Moreover, the Distance per Unit Time for the FFO (with
- conditions OFF) is large and stands at 2.95E+06, shadowing those of Cuckoo Search and Harmony
- 958 Search, which report 3.41E+01 and 2.85E+03, respectively. This metric highlights the FFO's
- 959 efficiency in covering large distances in the search space per unit of time, reinforcing its utility in
- 960 expansive and complex problem spaces where speed and breadth of exploration are paramount. It
- is quite clear that the FFO ranked well in all metrics (see Fig. 6).





Fig. 6 Ranking of algorithms in 20D and 50D settings

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964	Table 4 Overall results for 20D and 50D settings
201	

Algorithm		Best	itness			Executio	ution Time (s) Distance metric Distance						
	mean	std	min	max	mean	std	min	max	mean	std	min	max	- per Unit Time
Ant Colony Optimization	7.61E+01	1.41E+02	9.04E-02	6.78E+02	9.42E+05	1.02E+07	-9.60E+02	1.48E+08	1.59E+03	4.57E+03	0.00E+00	3.20E+04	2.09E+01
Bat Algorithm	4.07E+00	8.43E+00	1.80E-02	7.49E+01	1.58E+04	7.26E+04	-9.77E+02	5.13E+05	9.75E+01	7.05E+02	0.00E+00	9.91E+03	2.39E+01
Biogeography- Based Optimization	1.06E+01	1.74E+01	7.30E-02	1.37E+02	1.22E+04	5.86E+04	-9.45E+02	4.44E+05	2.69E+04	9.81E+04	0.00E+00	1.24E+06	2.55E+03
Cuckoo Search	1.20E+02	3.66E+02	5.37E-02	3.63E+03	4.22E+03	2.62E+04	-9.60E+02	2.99E+05	4.10E+03	1.23E+04	0.00E+00	9.31E+04	3.41E+01
FFO (additional conditions OFF)	5.35E+00	1.00E+01	2.62E-02	8.16E+01	2.88E+03	2.19E+04	-9.60E+02	2.70E+05	1.58E+07	6.46E+07	1.50E+03	6.49E+08	2.95E+06
FFO (additional conditions ON)	1.81E-01	2.90E-01	0.00E+00	2.67E+00	9.01E+03	4.40E+04	-9.02E+02	3.41E+05	5.80E+05	1.95E+06	0.00E+00	1.78E+07	3.21E+06
Firefly Algorithm	2.93E+02	7.74E+02	1.36E-01	7.35E+03	1.03E+04	5.17E+04	-9.48E+02	5.16E+05	2.06E+03	9.80E+03	0.00E+00	1.02E+05	7.03E+00
Flower Pollination Algorithm	4.43E+00	8.81E+00	2.39E-02	7.74E+01	4.27E+02	2.82E+03	-9.60E+02	4.03E+04	8.12E+02	2.66E+03	3.89E-02	2.16E+04	1.83E+02
Genetic Algorithm	3.50E+00	5.87E+00	2.41E-02	4.73E+01	4.01E+02	2.17E+03	-9.60E+02	1.86E+04	9.72E+01	2.11E+02	0.00E+00	1.95E+03	2.78E+01
Grey Wolf Optimizer	8.40E+00	1.72E+01	4.20E-02	1.52E+02	7.23E+02	3.46E+03	-9.60E+02	2.09E+04	3.10E+04	1.05E+05	6.21E-02	8.51E+05	3.68E+03
Harmony Search	1.83E+00	4.06E+00	6.99E-03	3.69E+01	4.15E+03	2.89E+04	-9.60E+02	3.48E+05	5.22E+03	2.10E+04	0.00E+00	2.12E+05	1.57E+02
Particle Swarm Optimization	3.19E+00	5.31E+00	2.50E-02	4.31E+01	4.80E+02	2.67E+03	-9.60E+02	3.35E+04	2.98E+05	9.34E+05	3.24E+01	1.05E+07	4.45E+04
Simulated Annealing	5.17E-02	7.16E-02	9.97E-04	5.57E-01	4.58E+03	3.05E+04	-8.21E+02	3.73E+05	5.18E+02	7.21E+02	1.78E+00	4.96E+03	2.86E+03
Tabu Search	1.40E+00	3.07E+00	3.91E-03	2.14E+01	7.04E+02	3.04E+03	-7.53E+02	2.18E+04	5.04E+02	5.25E+02	2.17E+01	2.04E+03	7.01E+03
Whale Optimization Algorithm	3.19E+00	5.25E+00	2.54E-02	4.18E+01	-5.35E+01	4.24E+02	-9.60E+02	6.14E+03	2.10E+04	6.20E+04	9.85E-02	5.91E+05	1.52E+03

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- Now, we gain further insights into the performance of all algorithms across the settings of 20D
- and 50D, as seen in Fig. 7. It can be seen that the FFO performs consistently similarly to other
- ⁹⁶⁸ algorithms at different settings of agents and iterations.







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Algorithm Performance Trends for Function Egg holder at Dimension 50



Fig. 7 Sample of trends across different experimental settings

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We revisit the ranking performance of the optimization algorithms across the longest (and 974 shortest) time to solve and that achieved the most (and least) accuracy. Similar to the case of 2D, 975 the Firefly algorithm leads this category with an appearance frequency of 75, indicating it often 976 requires the most time to solve the selected benchmark functions. On the other hand, the Ant Colony 977 Optimization appears far less frequently with a count of 6, suggesting a quicker resolution time but 978 possibly at the expense of other performance metrics like accuracy or search depth. Then, the 979 Simulated Annealing dominates the fastest to solve category and hence demonstrates its ability to 980 swiftly find solutions. In terms of accuracy, Cuckoo Search stands out with 22 appearances, 981 followed by the FFO (additional conditions OFF) and Particle Swarm Optimization, each scoring 13 982 occurrences. The Ant Colony Optimization leads as the least accurate with 29 appearances, 983 followed by the Bat Algorithm at 19, suggesting these algorithms might prioritize exploration or 984 speed over precision. 985

Figure 8 paints a series of comparisons between selected algorithms. Figure 8a compares the 986 average history fitness across all functions and algorithms. This graph shows the average fitness 987 history across iterations for various optimization algorithms. The FFO (additional conditions OFF) 988 starts with a rapid convergence compared to other algorithms, which is particularly noticeable 989 against the backdrop of more gradual improvements shown by the Genetic Algorithm and 990 Simulated Annealing. The FFO (additional conditions OFF) demonstrates a stabilization around 200 991 iterations, where its fitness value flatlines. This early convergence suggests that, on average, this 992 algorithm is efficient in quickly finding a promising area of the search space. 993

While Fig. 8b compares the best fitness achieved in FFO, Tabu Search, the Bat, and the Grey Wolf 994 Optimizer algorithms. In Fig. 8b, the performance of FFO (additional conditions OFF) is plotted 995 across various scalable functions under different settings, showing stable performance for all 996 functions, except the Rosenbrock, and especially under conditions of higher iterations and larger 997 agent numbers. These spikes indicate that FFO (additional conditions OFF) can excel in complex, 998 multimodal landscapes but show variable performance dependent on the function's characteristics 999 and the search space complexity. Comparing this performance to other algorithms, such as Tabu 1000 Search, Bat, and Grey Wolf Optimizer, across the same benchmark functions shows generally stable 1001 performance as well, with some exceptions where fitness spikes, similar to the behavior seen in 1002 FFO. Figure 8c shows the performance of all algorithms in tackling continuous and non-continuous 1003 scalable benchmarking functions. The FFO (additional conditions OFF) shows competitive 1004 performance in all function types. These figures reinforce the performance of the newly proposed 1005 FFO algorithm against notable algorithms. 1006

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1014 Fig. 8 Further cross examination between the FFO and other notable algorithms

The Friedman and Wilcoxon non-parametric statistical tests for ranking the above algorithm were again carried out for this leg of the investigation. Figure 9 shows that the FFO ranks 5 and 6.5 in

¹⁰¹⁷ 2D and 6 and 9.5 for 50D for these tests, respectively. PSO, GWO, FPA, and WOA rank in the top

spots. As seen above, FFO always ranks better without constraints (i.e., conditions = off).

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Fig. 9 Friedman and Wilcoxon non-parametric statistical tests carried out for higher dimensions (note: Top: 20D, Bottom: 50D)

1023 Additional testing on functions from the CEC benchmarks (at 2D, 20D and 50D settings)

1024 The IEEE Congress on Evolutionary Computation (IEEE CEC) is an annual conference that 1025 focuses on the latest developments and research in evolutionary computation. As part of this

1026 conference, benchmark functions are commonly used to evaluate the performance of optimization

algorithms. These benchmark functions, often called CEC functions, are specifically designed to
 present various challenges to optimization methods, such as multimodality, non-separability, and

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ruggedness. The CEC benchmark suites are updated periodically, and we examine some of the
functions that appear in the 2020 edition. Each function typically represents a specific optimization
problem with known difficulty, allowing for a comprehensive evaluation of an algorithm's
strengths and weaknesses across various problem landscapes. Table 5 lists the CEC 2020
functions; additional details can be found in the CEC report [66].

1034 Table 5 CEC 2020 functions.

No.	Functions	Fi *= Fi(x *)	Dimensions tested
1	Shifted and Rotated Bent Cigar Function (also, CEC 2017 F1)	100	2, 20, 50
2	Shifted and Rotated Schwefel's Function (also, CEC 2014 F11)	1100	2, 20, 50
3	Shifted and Rotated Lunacek bi-Rastrigin Function (also, CEC 2017 F7)	700	2, 20, 50
4	Expanded Rosenbrock's plus Griewangk's Function (also, CEC 2017 F19)	1900	2, 20, 50
5	Hybrid Function 1 (N=3) (also, CEC 2014 F17)	1700	20, 50
6	Hybrid Function 2 (N=4) (also, CEC 2017 F16)	1600	20, 50
7	Hybrid Function 3 (N=5) (also, CEC 2014 F21)	2100	20, 50
8	Composition Function 1 (N=3) (also, CEC 2017 F22)	2200	2, 20, 50
9	Composition Function 2 (N=4) (also, CEC 2017 F24)	2400	2, 20, 50
10	Composition Function 3 (N=5) (also, CEC 2017 F25)	2500	2, 20, 50

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The outcome of this analysis is listed in Table 6. This table shows that both FFO and PSW ranked first in terms of overall ranking among all metrics. In addition, Fig. 10 shows that the best version of FFO ranked between 3 and 6 for the Friedman and Wilcoxon non-parametric statistical tests, respectively. Overall, PSO, GA, and CS achieved the best rankings in this experiment. Please cite this paper as:

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1040 Table 6 Overall results for 2D, 20D, and 50D settings

		Best I	Fitness			Executio	n Time (s)			Distance per			
Algorithm	mean	std	min	max	mean	std	min	max	mean	std	min	max	Unit Time
						2D							
Ant Colony Optimization	1.30E+08	3.57E+08	1.43E+03	1.20E+09	1.15E+01	1.58E+01	1.27E-01	6.76E+01	7.90E+00	3.60E+01	0.00E+00	2.00E+02	6.86E-01
Bat Algorithm	4.82E+06	3.55E+07	8.03E+01	2.82E+08	9.10E+00	1.45E+01	2.70E-02	6.32E+01	1.64E+01	4.79E+01	0.00E+00	2.68E+02	1.80E+00
Biogeography-Based Optimization	6.95E+07	4.58E+08	4.34E+02	3.56E+09	1.79E+01	2.67E+01	9.80E-02	1.12E+02	5.47E+03	1.13E+04	0.00E+00	7.82E+04	3.05E+02
Cuckoo Search	1.37E+03	9.97E+02	-3.23E+01	2.91E+03	3.36E+02	6.73E+02	1.27E-01	3.07E+03	9.79E+01	9.71E+01	1.11E+00	4.88E+02	2.91E-01
FFO (additional conditions OFF)	9.30E+04	7.24E+05	-3.24E+01	5.75E+06	1.33E+01	2.30E+01	6.80E-02	1.01E+02	4.13E+06	7.28E+06	8.82E+04	4.05E+07	3.10E+05
FFO (additional conditions ON)	5.78E+05	4.49E+06	2.99E+01	3.56E+07	4.71E-01	6.22E-01	1.90E-02	4.02E+00	3.28E+05	3.09E+05	3.33E+04	1.74E+06	6.97E+05
Firefly Algorithm	2.49E+07	1.89E+08	-2.72E+01	1.50E+09	7.20E+02	1.42E+03	2.66E-01	6.58E+03	2.99E+02	4.82E+02	0.00E+00	2.29E+03	4.16E-01
Flower Pollination Algorithm	1.49E+03	1.03E+03	2.40E+01	2.92E+03	9.40E+00	1.47E+01	3.20E-02	6.43E+01	7.75E+01	6.90E+01	5.10E+00	2.78E+02	8.25E+00
Genetic Algorithm	1.89E+03	1.04E+03	2.13E+02	4.96E+03	5.04E+00	8.00E+00	3.10E-02	3.69E+01	1.44E+01	1.83E+01	1.37E-01	1.03E+02	2.86E+00
Grey Wolf Optimizer	2.37E+03	1.54E+03	2.13E+02	5.63E+03	1.87E+01	2.96E+01	6.70E-02	1.28E+02	6.63E+03	8.23E+03	9.07E+01	3.49E+04	3.55E+02
Harmony Search	1.56E+05	8.66E+05	1.60E+02	6.20E+06	4.31E+00	6.99E+00	1.60E-02	3.03E+01	8.13E+01	9.96E+01	0.00E+00	6.73E+02	1.88E+01
Particle Swarm Optimization	1.72E+03	1.12E+03	-3.24E+01	5.62E+03	5.63E+00	8.73E+00	4.20E-02	4.06E+01	1.66E+05	5.65E+05	1.82E+03	3.82E+06	2.95E+04
Simulated Annealing	5.52E+08	2.81E+09	1.51E+02	1.59E+10	9.67E-02	1.11E-01	1.03E-03	3.79E-01	9.49E+01	6.39E+01	2.27E+01	2.69E+02	9.81E+02
Tabu Search	6.58E+08	3.17E+09	1.11E+02	1.87E+10	2.21E-01	2.35E-01	1.10E-02	7.94E-01	5.77E+02	5.14E+02	2.89E+01	1.51E+03	7.01E+03
Whale Optimization Algorithm	2.51E+04	1.79E+05	2.24E+02	1.42E+06	5.65E+00	8.40E+00	3.70E-02	3.59E+01	3.73E+03	1.30E+04	2.94E-02	9.68E+04	1.52E+03
		-	-		-	20D	-			-			
Ant Colony Optimization	2.65E+09	8.24E+09	4.62E+02	3.57E+10	5.11E+01	6.37E+01	3.71E-01	2.23E+02	1.85E+03	8.06E+02	0.00E+00	3.45E+03	3.62E+01
Bat Algorithm	6.61E+09	1.89E+10	-2.62E+03	8.31E+10	1.03E+01	1.60E+01	1.64E-02	7.78E+01	5.03E+01	2.72E+02	0.00E+00	2.14E+03	4.87E+00
Biogeography-Based Optimization	5.12E+09	1.64E+10	2.55E+03	9.25E+10	1.87E+01	2.66E+01	4.89E-02	1.14E+02	4.00E+04	5.07E+04	0.00E+00	2.41E+05	2.14E+03
Cuckoo Search	2.19E+08	1.22E+09	-3.52E+03	8.52E+09	3.86E+02	7.26E+02	6.76E-02	3.29E+03	3.46E+03	1.79E+03	6.28E+02	8.95E+03	8.96E+00
FFO (additional conditions OFF)	1.09E+09	5.71E+09	-5.30E+03	4.41E+10	1.12E+01	1.67E+01	2.69E-02	7.25E+01	9.78E+06	1.10E+07	3.31E+05	4.18E+07	8.73E+05
FFO (additional conditions ON)	4.14E+09	1.36E+10	1.61E+03	8.26E+10	4.58E-01	4.52E-01	8.18E-03	2.13E+00	1.08E+06	8.76E+05	1.14E+05	4.05E+06	2.35E+06
Firefly Algorithm	7.10E+09	2.00E+10	7.12E+02	9.67E+10	7.83E+02	1.46E+03	1.46E-01	6.43E+03	2.26E+03	4.21E+03	0.00E+00	2.21E+04	2.89E+00
Flower Pollination Algorithm	1.13E+08	9.81E+08	-3.44E+03	9.29E+09	1.03E+01	1.55E+01	1.65E-02	7.08E+01	1.36E+03	5.65E+02	4.54E+02	2.80E+03	1.32E+02
Genetic Algorithm	1.24E+09	7.11E+09	-1.72E+03	5.63E+10	6.42E+00	9.13E+00	2.55E-02	4.11E+01	4.70E+02	4.02E+02	5.16E+01	1.62E+03	7.32E+01
Grey Wolf Optimizer	7.37E+08	3.09E+09	-9.46E+02	1.93E+10	2.06E+01	3.06E+01	3.25E-02	1.32E+02	4.25E+04	3.87E+04	3.35E+03	1.30E+05	2.06E+03
Harmony Search	1.21E+09	5.96E+09	-4.65E+03	4.49E+10	5.16E+00	7.54E+00	1.69E-02	3.30E+01	2.85E+03	2.56E+03	0.00E+00	1.50E+04	5.53E+02
Particle Swarm Optimization	2.97E+08	1.81E+09	-1.68E+03	1.54E+10	6.06E+00	8.65E+00	9.51E-03	3.73E+01	1.21E+05	1.79E+05	1.01E+04	1.24E+06	1.99E+04
Simulated Annealing	4.98E+09	2.48E+10	-1.65E+03	1.71E+11	1.05E-01	1.14E-01	0.00E+00	4.56E-01	8.90E+02	7.58E+02	9.54E+00	2.63E+03	8.44E+03
Tabu Search	3.90E+09	1.64E+10	-1.89E+03	1.05E+11	2.25E+00	2.34E+00	2.06E-02	7.89E+00	5.84E+02	5.05E+02	5.60E+01	1.77E+03	2.60E+02
Whale Optimization Algorithm	3.11E+09	9.79E+09	1.58E+03	4.70E+10	5.91E+00	8.41E+00	1.57E-02	3.61E+01	1.84E+04	1.57E+04	6.51E+02	5.97E+04	3.12E+03
						50D							
Ant Colony Optimization	1.85E+10	5.18E+10	7.57E+03	2.09E+11	1.18E+02	1.45E+02	1.45E+00	5.02E+02	3.26E+03	1.34E+03	5.48E+02	6.52E+03	2.77E+01
Bat Algorithm	3.69E+10	8.22E+10	-1.85E+03	3.27E+11	1.05E+01	1.55E+01	3.36E-02	8.09E+01	6.49E+01	2.63E+02	0.00E+00	1.84E+03	6.21E+00
Biogeography-Based Optimization	2.92E+10	6.74E+10	7.83E+03	2.67E+11	2.08E+01	2.93E+01	8.32E-02	1.50E+02	6.94E+04	8.48E+04	0.00E+00	3.52E+05	3.33E+03
Cuckoo Search	3.77E+09	1.99E+10	-6.37E+02	1.32E+11	3.94E+02	7.39E+02	9.99E-02	4.15E+03	8.57E+03	4.31E+03	2.15E+03	2.48E+04	2.17E+01
FFO (additional conditions OFF)	9.99E+09	3.85E+10	-1.74E+03	2.37E+11	1.19E+01	1.77E+01	4.70E-02	9.02E+01	1.66E+07	1.96E+07	5.24E+05	8.41E+07	1.39E+06
FFO (additional conditions ON)	2.45E+10	6.20E+10	8.85E+03	2.31E+11	5.04E-01	4.91E-01	1.42E-02	2.27E+00	1.71E+06	1.32E+06	1.81E+05	5.88E+06	3.39E+06
Firefly Algorithm	3.71E+10	7.91E+10	8.31E+03	2.75E+11	8.20E+02	1.53E+03	2.12E-01	8.64E+03	2.75E+03	6.27E+03	0.00E+00	4.05E+04	3.35E+00
Flower Pollination Algorithm	9.95E+08	6.53E+09	-1.77E+03	5.87E+10	1.08E+01	1.61E+01	3.27E-02	8.78E+01	3.30E+03	1.33E+03	3.62E+02	6.91E+03	3.05E+02
Genetic Algorithm	1.03E+10	4.35E+10	-3.59E+03	3.07E+11	7.94E+00	1.09E+01	4.99E-02	5.24E+01	1.08E+03	9.81E+02	1.00E+02	3.82E+03	1.36E+02
Grey Wolf Optimizer	3.80E+09	1.38E+10	2.07E+03	8.35E+10	2.15E+01	3.22E+01	7.34E-02	1.71E+02	6.80E+04	6.53E+04	5.60E+03	2.49E+05	3.15E+03
Harmony Search	9.83E+09	3.29E+10	-7.79E+03	2.09E+11	5.66E+00	7.96E+00	4.68E-02	4.25E+01	5.24E+03	4.16E+03	0.00E+00	2.22E+04	9.26E+02
Particle Swarm Optimization	2.04E+09	1.02E+10	-2.89E+03	7.03E+10	6.36E+00	9.28E+00	3.11E-02	4.93E+01	2.78E+05	6.00E+05	1.68E+04	5.13E+06	4.37E+04
Simulated Annealing	2.35E+10	7.58E+10	1.72E+03	3.82E+11	1.09E-01	1.24E-01	0.00E+00	5.94E-01	2.07E+03	1.84E+03	4.15E+01	6.29E+03	1.91E+04
Tabu Search	3.13E+10	8.78E+10	-3.73E+03	3.94E+11	5.59E+00	6.11E+00	6.74E-02	2.30E+01	8.88E+02	9.18E+02	6.63E+01	2.73E+03	1.59E+02
Whale Optimization Algorithm	1.34E+10	3.27E+10	8.56E+03	1.30E+11	6.11E+00	8.77E+00	1.22E-02	4.60E+01	2.97E+04	2.56E+04	8.29E+02	8.30E+04	4.85E+03

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1052 *Real engineering problems*

The CEC benchmark suit also contains several constraints and real engineering problems 1053 commonly used in the benchmarking analysis. As such, five additional engineering problems were 1054 examined herein. Table 6 lists these problems and their characteristics. It should be noted that the 1055 mathematical derivation for these problems is not included herein for brevity, yet it can be easily 1056 accessible from the original source of CEC 2020, as well as in [67]. The Friedman and Wilcoxon 1057 non-parametric statistical tests were carried out for each problem individually. Each problem was 1058 run with the following settings: Agents [25, 50, 100] and iterations [25, 100, 100]. Table 7 and 1059 Fig. 11 show the outcome of this analysis as well as a comparative history run for the three-bar 1060 truss design problem. Overall, it can be seen that the proposed algorithm has a decent performance, 1061 ranking within the top 5 spots in each problem (and first on the distance covered), with its best 1062 performance recorded on the three-bar truss design problem. This analysis also shows the need to 1063 improve further and tune FFO to allow it to enhance its performance on real engineering problems. 1064

1065 Table 6 CEC 2020 engineering problems.

No.	Functions	No. of constraints	Objective/Remark
10	Process flow sheeting problem	3	Minimize the flow sheeting process
17	Tension/Compression spring design	4	Optimize the weight of a spring
18	Pressure vessel design	4	Optimize the welding cost, material, and forming
19	Three-bar truss design problem	3	Minimize the weight of the bar structures

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1067 Table 7 Results on engineering problems (Ranks are based on fitness)

		Maxitora	tions: 25 Agents: 25			May I	torations: 100 Agents: 50			May I	orations: EQ0 Agents: 10	0
Algorithm	Bank	Rost Eitnoss, moon	Evecution Time mean	Total Distance mean	Pank	NIAX I Bost Eitnoss moon	Evocution Time mean	Total Dictance mean	Pank	Rost Eitnoss moon	Erations: 500, Agents: 10	
BA	11	0.0351	Execution nine_mean		11	0.0714			13	966229 5285		TOL
BRO	12	0.0331	12,4000	204 4217	- 11	0.0714	1 1500	206 2512	13	900229.3283	0.0900	
CS CS	5	0.0130	283 8200	0.9638	7	0.0252	13 4600	16 7597	7	0.0347	1 1400	
EEQ (conditions OEE)	2	0.0127	6 5400	104755 4358	6	0.0157	0.6100	24247 5107	8	0.0348	0.0800	
FFO (conditions ON)	8	0.0127	0.5400	22753 4404	8	0.0167	0.2900	9874 0739	6	0.0348	0.0800	
FA	4	0.0132	592 9600	15 4207	4	0.0140	27 9600	15 5375	4	0.0150	2 0400	
FPA	3	0.0127	6 5900	5 8156	2	0.0127	0 5800	6 8138	3	0.0140	0.0800	
GA	7	0.0145	3.2400	10.7102	5	0.0157	0.3100	3.3469	10	55481.1184	0.0500	
GWO	1	0.0127	12.5700	644.1331	3	0.0131	1.1400	150.7236	2	0.0138	0.2100	
PSW	9	0.0156	3.7200	2870.0385	1	0.0127	0.3600	1362.2240	1	0.0135	0.0500	
SA	6	0.0133	0.0300	11.0837	13	20016.2994	0.0100	3.7422	11	443221.9830	0.0000	
TS	13	2763.4846	0.1000	179.8630	12	7800.8091	0.0200	29.3575	9	0.2461	0.0100	
WOA	10	0.0169	3.9800	643.2659	10	0.0270	0.3600	181.7931	5	0.0161	0.0500	
		·				Pressure vessel de	sign					
Algorithm		Max Itera	tions: 25, Agents: 25			Max I	terations: 100, Agents: 50			Max II	erations: 500, Agents: 10	0
Algorithm	Rank	Best Fitness_mean	Execution Time_mean	Total Distance_mean	Rank	Best Fitness_mean	Execution Time_mean	Total Distance_mean	Rank	Best Fitness_mean	Execution Time_mean	Tota
BA	10	27672.8778	0.0700	0.0141	11	30169.8352	0.5100	0.1832	8	8101.1895	5.6300	
BBO	8	20177.3138	0.1400	651.7029	10	19536.3625	1.0900	0.0000	13	13435.2796	11.5400	
CS	3	8588.6665	0.8700	297.5321	1	6255.4730	12.0500	380.9517	2	6055.6658	250.7500	
FFO (conditions OFF)	6	16238.2724	0.0900	79738.3119	7	13338.4993	0.5500	528538.3793	3	6058.6276	6.0300	3
FFO (conditions ON)	7	19499.2357	0.0900	71257.3334	9	14521.7170	0.2600	266136.4509	10	10273.0019	0.8200	
FA	11	57102.8448	1.8100	0.0000	8	13490.6739	25.3300	62.4217	12	12401.2549	541.9300	
FPA	5	15354.0109	0.0700	39.0251	2	6384.4138	0.5900	229.8373	4	6082.2291	5.8500	
GA	9	21774.6840	0.0400	14.9825	6	11447.9394	0.2900	37.1125	5	6195.9443	2.9900	
GWO	2	7770.6504	0.1300	437.6659	4	7325.6852	1.0400	1456.1620	1	6049.9952	11.2400	
PSW	1	6093.6822	0.0500	10362.2512	3	6767.6893	0.3300	22045.4224	6	7330.6288	3.5100	
SA	13	853026517754487000.0000	0.0000	20.7782	13	277802.5589	0.0100	85.2185	11	11584.2557	0.0300	
TS	12	197045.3629	0.0100	13.2349	12	132269.8417	0.0300	58.0393	7	7336.3953	0.1800	
WOA	4	12478.3154	0.0400	212.7318	5	10313.6550	0.3400	972.9236	9	9937.7694	3.6700	
							problem					
		Maxitora	tions: 25 Agonts: 25			Inree-bar truss design	problem			May I	iorations: 500 Agonts: 10	<u></u>
Algorithm	Bank	Max Itera	tions: 25, Agents: 25	Total Distance mean	Rank	Max l	problem terations: 100, Agents: 50 Execution Time, mean	Total Dictance, mean	Rank	Max II Best Eitness mean	erations: 500, Agents: 10	0
Algorithm	Rank	Max Itera Best Fitness_mean	tions: 25, Agents: 25 Execution Time_mean	Total Distance_mean	Rank	Max I Max I Best Fitness_mean	problem terations: 100, Agents: 50 Execution Time_mean	Total Distance_mean	Rank	Max It Best Fitness_mean	erations: 500, Agents: 10 Execution Time_mean	0 Tota
Algorithm BA BBO	Rank 11 2	Max Itera Best Fitness_mean 275.5093 264.1611	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100	Total Distance_mean 0.0113 0.0000	Rank 3	Max I Max I Best Fitness_mean 263.9097 265.4840	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100	Total Distance_mean 0.0012 0.0000	Rank 4	Max It Best Fitness_mean 263.8950 264.8880	erations: 500, Agents: 10 Execution Time_mean 10.1600	0 Tota
Algorithm BA BBO CS	Rank 11 2 6	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4 9800	Total Distance_mean 0.0113 0.0000 0.6687	Rank 3 10 5	Max 1 Best Fitness_mean 263.9097 265.4840 264.5199	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400	Total Distance_mean 0.0012 0.0000 0.4967	Rank 4 8	Max It Best Fitness_mean 263.8950 264.8880 264.1509	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200	0 Tota
Algorithm BA BBO CS FFO (conditions OFF)	Rank 11 2 6 3	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700	Total Distance_mean 0.0113 0.0000 0.6687 284.0285	Rank 3 10 5 4	Max 1 Max 1 Best Fitness_mean 263.9097 265.4840 264.5199 264.0866	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764	Rank 4 8 6 2	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300	0 Tota
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON)	Rank 11 2 6 3 5	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034	Rank 3 10 5 4 6	Max I Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.6600	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980	Rank 4 8 6 2 5	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500	
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA	Rank 11 2 6 3 5 1	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 264.0344	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640	Rank 3 10 5 4 6 2	Max I Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.6600 263.9003	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236	Rank 4 8 6 2 5 3	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100	0 Tota
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FPA	Rank 11 2 6 3 5 1 8	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 266.2357 264.0344 269.5027	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339	Rank 3 10 5 4 6 2 7	Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.6600 263.9003 264.6662	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294	Rank 4 8 6 2 5 3 7	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 263.8926 264.2540	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400	
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FPA GA	Rank 11 2 6 3 5 1 8 7	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 266.2357 264.0344 269.5027 268.8947	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072	Rank 3 10 5 4 6 2 7 8	Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.6600 263.9003 264.6662 264.6835	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.4600	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142	Rank 4 8 6 2 5 3 7 9	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 263.8926 264.2540 265.1644	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600	
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA FPA GA GWO	Rank 11 2 6 3 5 1 8 7 12	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 266.2357 264.0344 269.5027 268.8947 282.8427	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157	Rank 3 10 5 4 6 2 7 8 12	Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.4600 1.8100	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466	Rank 4 8 6 2 5 3 7 9 13	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 263.8926 264.2540 265.1644 282.8402	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800	
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA FPA GA GWO PSW	Rank 11 2 6 3 5 1 8 7 12 4	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1600	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812	Rank 3 10 5 4 6 2 7 8 12 1	Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427 263.8915	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.9300 0.4600 1.8100 0.5400	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 172.3326	Rank 4 8 6 2 5 3 7 9 13 1	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 264.2540 265.1644 282.8402 263.8915	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800 5.9400	0 Tota 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA GA GWO PSW SA	Rank 11 2 6 3 5 1 8 7 12 4 12	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990 282.8427	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1600 0.0000	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812 3.2932	Rank 3 10 5 4 6 2 7 8 12 12 1	Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427 263.8915 282.8427	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.9300 0.4600 1.8100 0.5400 0.0100	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 172.3326 3.3107	Rank 4 8 6 2 5 3 7 9 13 1 12	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 264.2540 265.1644 282.8402 263.8915 263.8915	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800 5.9400 0.0500	0 Tota 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA GA GWO PSW SA TS	Rank 11 2 6 3 5 1 8 7 12 4 12 10	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990 282.8427 265.8990	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1600 0.0000 0.0100	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812 3.2932 4.2519	Rank 3 10 5 4 6 2 7 8 12 12 12 12 12	Infee-oar truss design Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427 263.8915 282.8427 282.8427	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.9300 0.4600 1.8100 0.5400 0.5400 0.0100 0.0200	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 172.3326 3.3107 20.3575	Rank 4 8 6 2 5 3 7 9 13 1 12 10	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 263.8919 263.8926 264.2540 265.1644 282.8402 263.8915 272.4788 267.1915	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800 5.9400 0.0500 0.1100	0 Tota
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA GA GWO PSW SA TS WOA	Rank 11 2 6 3 5 1 8 7 12 4 12 10 9	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990 282.8427 272.5299 270.1763	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1600 0.0000 0.0100 0.4000	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812 3.2932 4.2519 1.0332	Rank 3 10 5 4 6 2 7 8 12 12 12 12 12 9	Infee-oar truss design Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427 263.8915 282.8427 282.8427 282.8427 282.8427 282.8427 282.8427 282.8427 282.8427 282.8427 282.8427 282.8427 282.8427	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.9300 0.4600 1.8100 0.5400 0.5400 0.0100 0.0200 0.6200	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 172.3326 3.3107 20.3575 3.1695	Rank 4 8 6 2 5 3 7 9 13 1 12 10 11	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 263.8926 263.8926 264.2540 265.1644 282.8402 263.8915 272.4788 267.1915 267.9062	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800 5.9400 0.0500 0.1100 5.9900	0 1 1 1 1 1 1 1 1 1 1 1 1 1
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA GA GWO PSW SA TS WOA	Rank 11 2 6 3 5 1 8 7 12 4 4 12 10 9	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990 282.8427 272.5299 270.1763	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1600 0.0000 0.0100 0.4000	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812 3.2932 4.2519 1.0332	Rank 3 10 5 4 6 2 7 8 12 12 12 12 12 9	Mice-oar truss design Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427 263.8915 282.8427 282.8427 264.7512 Process flow sheeting	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.9300 0.4600 1.8100 0.5400 0.5400 0.0100 0.0200 0.6200 problem	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 172.3326 3.3107 20.3575 3.1695	Rank 4 8 6 2 5 3 7 9 13 1 12 10 11	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 264.2540 265.1644 282.8402 263.8915 272.4788 267.1915 267.9062	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800 5.9400 0.0500 0.1100 5.9900	0 Tota
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA FPA GA GA GWO PSW SA SA TS WOA	Rank 11 2 6 3 5 1 8 7 12 4 12 4 12 10 9	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990 282.8427 272.5299 270.1763 Max Itera	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1600 0.0100 0.0100 0.4000 tions: 25, Agents: 25	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812 3.2932 4.2519 1.0332	Rank 3 10 5 4 6 2 7 8 12 12 12 12 12 9	Incee-bar truss design Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427 263.8915 282.8427 264.7512 Process flow sheeting Max I	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.4600 1.8100 0.5400 0.0100 0.0200 0.0200 0.6200 problem terations: 100, Agents: 50	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 172.3326 3.3107 20.3575 3.1695	Rank 4 8 6 2 5 3 7 9 13 1 12 10 11	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 263.8926 263.8926 265.1644 282.8402 263.8915 263.8915 267.4788 267.1915 267.9062 Max It	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800 5.9400 0.0500 0.1100 5.9900 erations: 500, Agents: 10	
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Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA FPA GA GA GWO PSW SA SA TS WOA MOA BA BBO CS FFO (conditions OFF)	Rank 11 2 6 3 5 1 8 7 12 4 12 10 9 Rank 12 13 6 8 0	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990 282.8427 272.5299 270.1763 Max Itera Best Fitness_mean 69813.0780 158420.4126 16.4045 667.6444	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1600 0.0000 0.0100 0.0100 0.0100 0.4000 tions: 25, Agents: 25 Execution Time_mean 0.3500 1.3600 5.7400 0.3500 0.3500 0.2700	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812 3.2932 4.2519 1.0332 Total Distance_mean 0.0000 8.6180 0.0989 744.3846 701 7025	Rank 3 10 5 4 6 2 7 8 12 12 12 12 12 9 Rank 12 13 5 7 2	Mice-bar truss design Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427 263.8915 282.8427 264.7512 Process flow sheeting Max I Best Fitness_mean 2711.2231 10149.9007 1.3677 1.4775	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.4600 1.8100 0.5400 0.0100 0.0200 0.6200 problem terations: 100, Agents: 50 Execution Time_mean 0.7600 1.4100 16.5000 0.7100	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 172.3326 3.3107 20.3575 3.1695 Total Distance_mean 0.2014 22.3039 0.8389 4498.6072 286.622	Rank 4 8 6 2 5 3 7 9 13 1 12 10 111 Rank 5 13 6 2 10	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 263.8926 263.8926 263.8915 265.1644 282.8402 263.8915 272.4788 267.1915 267.9062 Max It Best Fitness_mean 1.2507 14285.1378 1.2510 1.2500	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800 5.9400 0.0500 0.1100 5.9900 erations: 500, Agents: 10 Execution Time_mean 6.8500 15.5600 325.9700 8.1900 1.0300	0 Tota 0 0 Tota
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA FPA GA GA GWO PSW SA SA SA SA TS WOA MOA BA BBO CS FFO (conditions OFF) FFO (conditions OFF)	Rank 11 2 6 3 5 1 8 7 12 4 12 10 9 Rank 12 13 6 8 9 2	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990 282.8427 272.5299 270.1763 Max Itera Best Fitness_mean 69813.0780 158420.4126 16.4045 667.6444 1542.7608	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1200 0.7400 0.0000 0.0100 0.0100 0.0100 0.0100 0.4000 tions: 25, Agents: 25 Execution Time_mean 0.3500 1.3600 5.7400 0.3500 0.2700 8.700	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812 3.2932 4.2519 1.0332 Total Distance_mean 0.0000 8.6180 0.0989 744.3846 701.7935	Rank 3 10 5 4 6 2 7 8 12 12 12 12 12 12 13 5 7 2 2 2	Mice-bar truss design Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427 263.8915 282.8427 264.7512 Process flow sheeting Max I Best Fitness_mean 2711.2231 10149.9007 1.3677 1.4775 1.2500	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.4600 1.8100 0.04600 1.8100 0.0200 0.0200 0.6200 problem terations: 100, Agents: 50 Execution Time_mean 0.7600 1.4100 16.5000 0.7100 0.5400	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 172.3326 3.3107 20.3575 3.1695 Total Distance_mean 0.2014 22.3039 0.8389 4498.6072 3816.3316 0.2027	Rank 4 8 6 2 5 3 7 9 13 1 12 10 111 Rank 5 13 6 2 100	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 263.8926 263.8926 263.8915 265.1644 282.8402 263.8915 272.4788 267.1915 267.9062 Max It Best Fitness_mean 1.2507 14285.1378 1.2510 1.2500 2.3309	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800 5.9400 0.0500 0.1100 5.9900 erations: 500, Agents: 10 Execution Time_mean 6.8500 15.5600 325.9700 8.1900 1.0200 716.7600	0 Tota 0 0 Tota
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA FPA GA GA GWO PSW SA SA SA TS WOA MOA Hagorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FFA	Rank 11 2 6 3 5 1 8 7 12 4 12 10 9 Rank 12 13 6 8 9 3 4	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990 282.8427 272.5299 270.1763 Max Itera Best Fitness_mean 69813.0780 158420.4126 16.4045 667.6444 1542.7608 1.2526	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1200 0.7400 0.1600 0.0000 0.0100 0.0100 0.0100 0.0100 0.4000 tions: 25, Agents: 25 Execution Time_mean 0.3500 1.3600 5.7400 0.3500 0.2700 8.8700 0.3200	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812 3.2932 4.2519 1.0332 Total Distance_mean 0.0000 8.6180 0.0989 744.3846 701.7935 0.1592 1.4238	Rank 3 10 5 4 6 2 7 8 12 12 12 12 12 5 7 8 7 8 7 2 3 6	Mice-bar truss design Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427 263.8915 282.8427 264.7512 Process flow sheeting 2711.2231 10149.9007 1.3677 1.4775 1.2500 1.2501 1.4234	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.4600 1.8100 0.4600 1.8100 0.5400 0.0100 0.0200 0.6200 problem terations: 100, Agents: 50 Execution Time_mean 0.7600 1.4100 1.6.5000 0.7100 0.5400 0.5400 0.7100 0.5400	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 172.3326 3.3107 20.3575 3.1695 Total Distance_mean 0.2014 22.3039 0.8389 4498.6072 3816.3316 0.2927 0.9705	Rank 4 8 6 2 5 3 7 9 13 1 12 10 111 Rank 5 13 6 2 100 3 6 2 100 3 0	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 263.8926 263.8926 263.8915 265.1644 282.8402 263.8915 272.4788 267.1915 267.9062 CMax It Best Fitness_mean 1.2507 14285.1378 1.2510 1.2500 2.3309 1.2500	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800 5.9400 0.0500 0.1100 5.9900 erations: 500, Agents: 10 Execution Time_mean 6.8500 15.5600 325.9700 8.1900 1.0200 716.7600 8.2700	0 Tota 0 0 Tota
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA GA GA GWO PSW SA SA SA TS WOA MOA Hagorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA GA	Rank 11 2 6 3 5 1 8 7 12 4 12 10 9 Rank 12 13 6 8 9 3 4 7	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990 282.8427 272.5299 270.1763 Max Itera Best Fitness_mean 69813.0780 158420.4126 16.4045 667.6444 1542.7608 1.2526 8.7331	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1200 0.7400 0.0100 0.0100 0.0100 0.0100 0.0100 0.0100 0.0100 0.3500 1.3600 5.7400 0.3500 0.2700 8.8700 0.2300 0.0200	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812 3.2932 4.2519 1.0332 Total Distance_mean 0.0000 8.6180 0.0989 744.3846 701.7935 0.1592 1.4028	Rank 3 10 5 4 6 2 7 8 12 12 12 12 12 5 7 2 33 6 9	Mice-bar truss design Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.0866 264.6600 263.9003 264.6662 264.6662 264.6835 282.8427 263.8915 282.8427 264.7512 Process flow sheeting Max I Best Fitness_mean 2711.2231 10149.9007 1.3677 1.4775 1.2500 1.2501 1.4384 8.7747	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.4600 1.8100 0.4600 1.8100 0.0200 0.0200 0.6200 problem terations: 100, Agents: 50 Execution Time_mean 0.7600 1.4100 1.6.5000 0.7100 0.5400 37.6700 0.6400 0.2700	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 1772.3326 3.3107 20.3575 3.1695 Total Distance_mean 0.2014 22.3039 0.8389 4498.6072 3816.3316 0.2927 0.9795	Rank 4 8 6 2 5 3 7 9 13 1 12 10 111 Rank 5 13 6 13 6 10 3 9 7	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 263.8926 263.8926 263.8915 265.1644 282.8402 263.8915 272.4788 267.1915 267.9062 267.9062 Max It Best Fitness_mean 1.2507 14285.1378 1.2510 1.2500 2.3309 1.2500 1.250 1.250 1.2500 1.2500 1.250 1.250 1.250 1.2500 1.25	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800 5.9400 0.0500 0.1100 5.9900 erations: 500, Agents: 10 Execution Time_mean 6.8500 15.5600 325.9700 8.1900 1.0200 716.7600 8.2700	0 Tota 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
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Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA FPA GA GWO PSW SA TS WOA Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions OFF) FFO (conditions OFF) FFO (conditions ON) FA GA GWO PSW SA TS	Rank 11 2 6 3 5 1 8 7 12 4 12 10 9 Rank 12 13 6 8 9 3 4 7 13 6 8 9 3 4 7 2 1 10	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990 282.8427 272.5299 270.1763 Max Itera Best Fitness_mean 69813.0780 158420.4126 16.4045 667.6444 1542.7608 1.2526 8.7331 56.1340 1.2509 1.2500 8200.0204 5914.7086	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1200 0.0100 0.0100 0.0100 0.0100 0.0100 0.4000 tions: 25, Agents: 25 Execution Time_mean 0.3500 1.3600 5.7400 0.3500 0.2700 8.8700 0.2300 0.2300 0.0900 0.8400 0.1300 0.0000 0.0100	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812 3.2932 4.2519 1.0332 Total Distance_mean 0.0000 8.6180 0.0989 744.3846 701.7935 0.1592 1.4028 1.3388 0.4131 114.8741 0.3118 1.9175	Rank 3 10 5 4 6 2 7 8 12 12 12 12 13 5 7 2 3 6 8 4 10 9	Mice-bar truss design Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427 263.8915 282.8427 264.7512 Process flow sheeting Max I Best Fitness_mean 2711.2231 10149.9007 1.3677 1.4775 1.2500 1.2501 1.4384 8.7747 1.2504 1.2500 26.0589 18.0536	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.4600 1.8100 0.4600 1.8100 0.0200 0.6200 problem terations: 100, Agents: 50 Execution Time_mean 0.7600 1.4100 1.6.5000 0.7100 0.5400 37.6700 0.6400 0.3700 1.2700 0.3900 0.0100 0.0200	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 172.3326 3.3107 20.3575 3.1695 Total Distance_mean 0.2014 22.3039 0.8389 4498.6072 3816.3316 0.2927 0.9795 0.3217 1.2805 177.3422 3.1196 13.5297	Rank 4 8 6 2 3 7 9 13 1 12 10 11 Rank 5 13 6 2 100 3 9 7 4 10 3 9 7 4 10 3 9 7 4 11 8	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 263.8926 263.8926 263.8926 263.8915 265.1644 282.8402 263.8915 272.4788 267.1915 267.9062 263.901 265.07 14285.1378 1.2507 14285.1378 1.2510 1.250 1.2500 1.2500 1.2500 1.250 1.2500 1	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 13.4800 5.9400 0.0500 0.1100 5.9900 erations: 500, Agents: 10 Execution Time_mean 6.8500 15.5600 325.9700 8.1900 1.0200 716.7600 8.2700 4.7700 14.3800 4.0700 0.0400 0.1500	
Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions ON) FA FA FPA GA GWO PSW SA TS WOA Algorithm BA BBO CS FFO (conditions OFF) FFO (conditions OFF)	Rank 11 2 6 3 5 1 8 7 12 4 12 10 9 Rank 12 13 6 8 9 3 4 12 13 6 8 9 3 4 12 13 6 8 9 3 4 12 13 6 8 9 3 4 7 2 1 10 5	Max Itera Best Fitness_mean 275.5093 264.1611 268.3153 264.3765 266.2357 264.0344 269.5027 268.8947 282.8427 265.8990 282.8427 272.5299 270.1763 Max Itera Best Fitness_mean 69813.0780 158420.4126 16.4045 667.6444 1542.7608 1.2526 8.7331 56.1340 1.2509 1.2500 8200.0204 5914.7086 14.0063	tions: 25, Agents: 25 Execution Time_mean 0.4100 0.8100 4.9800 0.2700 0.2600 9.1500 0.5800 0.1200 0.7400 0.1200 0.0100 0.0100 0.0100 0.0100 0.3500 1.3600 5.7400 0.3500 0.2700 8.8700 0.2300 0.	Total Distance_mean 0.0113 0.0000 0.6687 284.0285 314.5034 0.2640 0.7339 0.0072 0.0157 75.3812 3.2932 4.2519 1.0332 Total Distance_mean 0.0000 8.6180 0.0989 744.3846 701.7935 0.1592 1.4028 1.3388 0.4131 114.8741 0.3118 1.9175 6.2861	Rank 3 10 5 4 6 2 7 8 12 12 12 12 13 5 7 2 33 6 8 4 10 9 110 9 11	Mice-bar truss design Max I Best Fitness_mean 263.9097 265.4840 264.5199 264.0866 264.0866 264.6600 263.9003 264.6662 264.6835 282.8427 263.8915 282.8427 264.7512 Process flow sheeting Max I Best Fitness_mean 2711.2231 10149.9007 1.3677 1.4775 1.2500 1.2501 1.4384 8.7747 1.2504 1.2500 26.0589 18.0536 1802.030	problem terations: 100, Agents: 50 Execution Time_mean 0.9600 1.8100 24.0400 0.8900 0.4200 53.0000 0.9300 0.4600 1.8100 0.0400 0.0100 0.0200 0.6200 problem terations: 100, Agents: 50 Execution Time_mean 0.7600 1.4100 1.6.5000 0.7100 0.5400 37.6700 0.6400 0.3700 1.2700 0.3900 0.0100 0.0200 0.0200 0.0200 0.5200	Total Distance_mean 0.0012 0.0000 0.4967 1761.7764 926.2980 0.2236 0.0294 0.0142 2.8466 172.3326 3.3107 20.3575 3.1695 V 0.2014 22.3039 0.8389 4498.6072 3816.3316 0.2927 0.9795 0.3217 1.2805 177.3422 3.1196 13.5297 6.9295	Rank 4 8 6 2 3 7 9 13 1 12 10 11 Rank 5 13 6 2 10 3 9 7 9 11 8 12	Max It Best Fitness_mean 263.8950 264.8880 264.1509 263.8919 264.1359 263.8926 264.2540 263.8926 264.2540 265.1644 282.8402 263.8915 272.4788 267.1915 267.9062 263.901 265.1378 1.2507 14285.1378 1.2510 1.250 1.2500 1.2500 1.2500 1.25	erations: 500, Agents: 10 Execution Time_mean 10.1600 19.1400 439.9200 10.4300 0.8500 952.0100 10.3400 4.8600 18.4800 5.9400 0.0500 0.1100 5.9900 erations: 500, Agents: 10 Execution Time_mean 6.8500 15.5600 325.9700 8.1900 1.0200 716.7600 8.2700 4.7700 14.3800 4.0700 0.0400 0.1500 4.6700	

al Distance_mean
0.0000
0.0000
3.8306
2769.1185
2896.5719
29.8529
5.9666
4.3979
61.2991
653.0028
2.5749
9.0047
47.2800
al Distance_mean
2.4383
16406.9804
461.4131
3550676.4451
796115.0139
123.3595
118.4741

21.8192	
11543.6784	
35351.4385	
167.9289	
226.9320	
10364.2569	

Distance_mean
0.0435
84.4678
0.4726
8258.9518
1930.4116
0.0785
0.2196
0.3835
3.3070
402.4285
4.3317
91.2797
2.2217

Distance_mean
0.5123
0.0000
0.4688
21269.7306
7224.1551
1.1939
0.6424
0.6688
4.8077
378.2226
3.4190
48.5607
25.7932

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Convergence History on Three-bar Truss Design - Dim: 2, Max Iter: 100, Agents: 50

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(d) Process flow sheeting problem

Fig. 11 Rankings on real engineering problems

1079 *Future research needs*

In order to further examine the performance of the proposed algorithm, we present a new set of results (see Fig. 12). In this analysis, we focus on function attributes like continuity, separability, multimodality, differentiability, and performance in continuous function optimization are demonstrated. The plots assess performance based on the average of metrics and visually display how each algorithm performs across these attributes under the specific settings.

This figure shows results produced by filtering for specific settings (Iterations=1000, Agents=100) 1085 and merging these results with predefined function attributes. These plots reveal distinct 1086 performance characteristics that generally show more modest peaks across the attributes, 1087 suggesting that some algorithms perform better in handling specific functions (i.e., Separable 1088 functions). For example, the FFO algorithm ranks first and fifth in continuous functions, first and 1089 1090 fourth in differentiable and non-differentiable, second and sixth on multimodal and nonmultimodal functions, fifth in scalable and non-scalable functions, and eighth and fifth on 1091 separable and non-separable functions. 1092

These insights, together with those gained from the testing on real engineering problems, can also 1093 provide technical improvement areas for FFO (as well as the selected algorithms). For example, 1094 FFO could focus on enhancing its performance in non-separable and multimodal landscapes, 1095 perhaps by integrating more adaptive search strategies or improving its handling of function 1096 differentiability through better derivative estimation or step size adjustment mechanisms. 1097 Additionally, efforts to boost scalability could be crucial, especially for handling higher-1098 dimensional optimization problems more effectively. In addition, there is a need to further examine 1099 the proposed algorithms against some of the recently developed algorithms (namely, SASS, 1100 COLSHADE, sCMAGES). At the moment, the performance of FFO can indeed be improved to 1101 match it with the aim of further enhancing it against the aforementioned leading algorithms. In 1102 addition, the authors hope that future editions from FFO include multi- and many-objective 1103 optimization capabilities. 1104

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1109 1110



1111 6.0 Conclusions

1112 The Firefighter optimization (FFO) algorithm offers a metaheuristic approach for optimization. This

algorithm was examined against 13 commonly used optimization algorithms and 38 functions

1114 (including benchmark functions, CEC 2020 standard functions, and real engineering problems).

Our results demonstrate that in 2D analysis, FFO ranks in the top 3 slots in terms of the best fitness

and space covered and ranks first in the Distance per Unit Time metric. Similarly, the performance

of the FFO algorithm also ranks third in higher dimensions (20D and 50D) and maintains a top 5

1118 performance across various metrics. Our analysis also indicates a few possible means to improve

the performance of FFO, primarily on scalable/non-scalable functions and separable/non-separable

1120 functions, as well as on performing on real optimization problems.

1121 Data Availability

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

1124 Firefighter optimization (FFO) can be accessed from [to be added].

1125 Conflict of Interest

1126 The authors declare no conflict of interest.

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